IMPLEMENTING PLANNING PROCEDURES FOR THE PROVISION OF DISCRETE PUBLIC GOODS*

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1. INTRODUCTION

The purpose of this paper is to investigate several alternative procedures for making collective decisions regarding the provision of discrete public goods. Although much attention has recently been focused on the design of procedures which elicit the true preferences of a population in order to make efficient decisions, we will show that each of those proposed possess undesirable properties given the nature of our problem.

Let us begin by summarizing the particular application that led us to this investigation. For each of the past three years, the Public Broadcasting System (PBS) has selected programs to be broadcast through a market process. This experimental market, called the Station Program Cooperative (SPC), enables each of the approximately 150 member stations to make purchase decisions out of an initial set of approximately 130 proposed programs. As far as the stations are concerned, each potential program is a public good in as much as the transmission costs of providing it to more than one station are assumed to be zero. While one might imagine that the quantity of a program provided might legitimately be varied over a certain range (e.g. by varying the number of episodes or the length of time of each episode), the nature of the present institution generally does not allow for these quantity adjustments. Thus we are confronted with a problem in the provision of discrete public goods where stations must make collective decisions to either accept or reject each program proposal.

We will proceed to discuss and analyze the existing mechanism along with several alternatives. As we shall see, each mechanism specifies a message space in which stations may communicate, a decision rule for accepting or rejecting each program, and a tax rule which levies lump-sum amounts on each station. The choice of message space is by no means an insignificant one. As we shall see, it may be the major shortcoming of the present SPC procedure since each station is confronted with prices for each program and may communicate only its willingness to buy or not buy.

We will require that each candidate procedure satisfy the following requirements:

1. The center may not run a deficit -- i.e., the total revenues collected from stations must be at least the total cost of all programs provided.
2. No station may go bankrupt -- i.e., the total amount collected from a station must not exceed its programming budget.
3. The bundle of programs produced must not be Pareto dominated by any other feasible bundle -- i.e., there is no other bundle which satisfies 1) and 2) such that each station is at least as well-off with this bundle and at least some station is strictly better off.

The existing mechanism will be analyzed in Section II and several alternatives which have been found in the literature will be examined in later sections. Unfortunately, our results are largely negative. For the current SPC procedure we will show that although 1) and 2) are guaranteed due to the nature of the price adjustment algorithm, an equilibrium may not exist and if it does it may not satisfy 3). Due to this we will then proceed to examine two other classes of procedures. In Section III, we will examine the mechanism suggested by Drèze and de la Vallee Poussin [2] and Malinvaud [10]. Under their mechanism it must be assumed that each station does not take the other stations' decisions as given, but must act as though the decisions of the other stations are those that are the least favorable to it. In other words, stations will be assumed to make "minimax" decisions. As they have shown, when the quantity of a public good provided may be varied continuously, this mechanism will lead stations to correctly reveal their preferences except at corner solutions. With discreteness, however, all solutions are corner solutions and we shall see that, in general, a station will not necessarily reveal its true preferences.

In Section IV, we will examine the class of mechanisms which are strongly individual incentive compatible. This class has been extensively studied previously by Green and Laffont [5,6,7] and Groves [7] for the case of a single discrete public good. If there is more than one such good, we will prove that there does not exist a mechanism satisfying 1), 2), and 3) for which each station has a dominant strategy. More specifically, our result points out the futility of a search for a mechanism which elicits the truthful willingness to pay for each public good from each member of the population. Due to this result we will proceed to discuss some weaker notions of incentive compatibility and related mechanisms in Section V.

II. THE SPC PROCEDURE

The existing SPC procedure may be summarized as follows (for further information see Perejohn and Noll [3]). At each stage of the process, each station is confronted with a price for each program and asked to indicate which programs it would be willing to purchase at these prices. Using these decisions the center calculates new prices such that if the purchase decisions of all stations remain unchanged, the total revenue collected on all produced programs will exactly cover their costs. The process reiterates until it converges.

At the heart of this mechanism is a formula for computing the cost share of a station on each program at each iteration. This share is based upon the programming budget of the station and the population of the area it serves relative to the budgets and the population served by all stations which indicated a willingness to
purchase the program on the previous iteration. More precisely, the price, \( p_j^i(t + 1) \), of program \( j \) to station \( i \) at time \( t + 1 \) is computed as

\[
p_j^i(t + 1) = \frac{\beta^i}{\sum_{k \neq i} b_j^k(t) + B^i} + \frac{2}{\sum_{k \neq i} n_j^k(t) + N^i} c_j
\]

where

- \( b_j^k \): the programming budget of station \( k \),
- \( n_j^k \): the population of the area served by station \( k \),
- \( c_j \): the cost of producing program \( j \),
- \( z_j^k(t) \): the purchase decision of station \( k \) at time \( t \) to accept \( (z_j^k(t) = 1) \) or reject \( (z_j^k(t) = 0) \) program \( j \).

We should be careful to note that the current procedure does allow for the exclusion of stations from use of a particular program. Specifically, if the process converges and a station has indicated it does not wish to purchase a program at its current cost share, then the station is not charged for that program and is excluded from using it.

As stated in the previous section, one of the major difficulties that arises here is that a station may only indicate its willingness to buy or not buy at a given set of prices. A station which is willing to pay some amount for a program which is less than the price it faces may only decline to purchase the program and has no way to communicate a bid. Neither does a station which is willing to pay more than the current price have any way to signal the intensity of its preference.

In addition to this difficulty this procedure has several other unfortunate features:

1. Equilibria need not be efficient.
2. There may be multiple equilibria which may be Pareto-ranked.
3. If there are multiple equilibria, the efficient equilibrium may be unstable.
4. No equilibrium may exist.

We can illustrate these points by means of two simple examples. Without loss of generality, we will use a simplified version of the price formula given above, namely

\[
p_j^i(t + 1) = \frac{b_j^1}{\sum_{k \neq i} b_j^k(t) + B^i} c_j
\]

**Example 1:**

Suppose there are two stations whose preferences are representable by the utility functions

\[
U^1(z_1^1, z_2^1, y_1) = 3z_1^1 + 2z_2^1 + .1y_1
\]

and

\[
U^2(z_1^2, z_2^2, y_2) = 2z_1^2 + 3z_2^2 + .1y_2
\]

where \( y_i \) is the amount of some private good consumed by station \( i \).

Each station is endowed with 5 units of the private good \( (b_1^1 = b_2^2 = 5) \) and may choose from a list of two programs each with a cost of five \( (c_1 = c_2 = 5) \). It may be verified that there are multiple equilibria in this example which are given by

\[
p^1 = (5,2.5) \quad \quad \quad p^2 = (2.5,5)
\]

\[
z^1 = (1,0) \quad \quad \quad z^2 = (0,1)
\]
and
\[ p^1 = (2.5, 2.5) \quad p^2 = (2.5, 2.5) \]
\[ z^1 = (1, 1) \quad z^2 = (1, 1) \]

The second equilibrium clearly Pareto dominates the first. It may also be shown that the efficient equilibrium is unstable. To see this consider the following prices:
\[ p^1(0) = (2.5, 2.5 + \varepsilon) \quad p^2(0) = (2.5, 2.5) \]

Iterating through the algorithm for calculating prices and demands will give the following cycle:
\[
\begin{align*}
  z^1(0) &= (1, 0) & z^2(0) &= (1, 1) \\
  p^1(1) &= (2.5, 2.5) & p^2(1) &= (2.5, 5) \\
  z^1(1) &= (1, 1) & z^2(1) &= (0, 1) \\
  p^1(2) &= (5, 2.5) & p^2(2) &= (2.5, 2.5) \\
  z^1(2) &= (1, 0) & z^2(2) &= (1, 1) \\
  p^1(3) &= (2.5, 2.5) = p^1(1) & p^2(3) &= (2.5, 5) = p^2(1)
\end{align*}
\]

Example 2:

To show that no equilibria may exist with this procedure, let
\[
U^1(z^1, z^2, y^1) = z^1_1 z^1_2 + z^1_1 + .1y^1
\]

and
\[
U^2(z^1, z^2, y^2) = \max(z^2_1, z^2_2) + .1y^2
\]

where \( B^1 = 8, B^2 = 8, C_1 = 6 \) and \( C_2 = 5 \). The driving force in this example is that station 2 will always choose only one program -- the one with the lowest price. On the other hand, station 1 will only choose program 2 when it can afford to also select program 1. This will occur only when station 2 has selected the first program, since when station 2 has chosen the second program, the first station is given the prices \( p^1_1 = 6 \) and \( p^2_1 = 2.5 \) and cannot afford to purchase both. On the other hand, when station 2 purchases the first program, the first station can afford to purchase both programs. But this action will cause the price of the second program to be less than the first to station 2 (\( p^2_1 = 3 \) and \( p^2_2 = 2.5 \)) and thus station 2 will revert to choosing program 2.

Assuming initial prices of \( p^1(0) = p^2(0) = (6, 5) \) and iterating through the algorithm gives the following sequences of prices and demands:
\[
\begin{align*}
  z^1(0) &= (1, 0) & z^2(0) &= (0, 1) \\
  p^1(1) &= (6, 2.5) & p^2(1) &= (3, 5) \\
  z^1(1) &= (1, 0) & z^2(1) &= (1, 0) \\
  p^1(2) &= (3, 5) & p^2(2) &= (3, 5) \\
  z^1(2) &= (1, 1) & z^2(2) &= (1, 0) \\
  p^1(3) &= (3, 5) & p^2(3) &= (3, 2.5) \\
  z^1(3) &= (1, 1) & z^2(3) &= (0, 1) \\
  p^1(4) &= (6, 2.5) & p^2(4) &= (3, 2.5) \\
  z^1(4) &= (1, 0) & z^2(4) &= (0, 1) \\
  p^1(5) &= (6, 2.5) = p^1(1) & p^2(5) &= (3, 5) = p^2(1)
\end{align*}
\]

Thus, with these initial prices, the algorithm will lead to a cycle. Furthermore, we assert by the above logic that with any other initial prices we will immediately generate a set of purchase decisions for both stations which are found at an iteration in the above cycle. Therefore, no stationary point can be found.
Because of the need to find a stable equilibrium within a small number of iterations, two additional rules have been adopted. First, programs which are not generating revenues to cover a predetermined fraction of their costs in a given iteration are dropped from consideration. This predetermined fraction rises as the number of iterations increase. Second, once all but a few programs that are likely to be purchased have been eliminated, the center announces that all stations will be required to continue selecting a program that they selected in the previous iteration if the price does not increase. These two rules do succeed in guaranteeing the convergence of the process at some loss of efficiency since outside of the context of the SPC rules, the final list of programs purchased may not even be an equilibrium.

One final drawback of this procedure should also be pointed out. Programs which generate little support in the early rounds of the process are likely to have their prices rise so rapidly that they never receive future support. As an extreme example of this, consider a program which is not purchased by any station in the first round. All stations will be confronted with a price equal to the full cost of that program in the second round and it is indeed unlikely that it will ever be purchased.

III. THE MINIMAX PROCEDURE

As an alternative mechanism to the existing procedure, we investigated the properties of a process adopted from Drèze and de la Vallee Poussin [2] and Malinvaud [10]. One of the virtues of this process, relative to the existing one, is that the message space in which each station may communicate is not as constrained. Instead of communicating purchase decisions at each stage of the planning procedure, each station is asked to communicate what it would be willing to pay for the introduction of an unproduced program, or, for a produced program, what compensation it would require to remain just as well off after the deletion of the program as it would have been without the deletion. On the other hand, as pointed out in the introduction, this procedure has one major drawback — namely, it requires that each station assume that the other stations' decisions are those which are the least favorable to it. However, ignoring this assumption for the moment, we will proceed to examine this process.

The planning mechanism we propose here is a mapping $F: \mathbb{R}^P \rightarrow \{0,1\}^P \times \mathbb{R}^S$ where the inputs to the planning procedure at each stage are vectors, $m^i(z)$, one for each station describing their willingness to pay for each program given that the system is currently in social state $z$. Here $z = (x,y) \in \{0,1\}^P \times \mathbb{R}^S$ is the social state where $x^i = x^i, i = 1, \ldots, P$, indicates whether or not program $i$ is being produced, and $y^i_\pi = y^i_\pi, i = 1, \ldots, S$, is the amount of the private good which station $i$ possesses in this state. Given an initial decision by the center, $z(0)$, the stations are asked to report the vector $(m^1(z(0)), \ldots, m^S(z(0)))$. The center computes $z(1)$, etc., according to the following rule.
Rule: Given $z(t)$ and $(m_1^t(z(t)), \ldots, m_S^t(z(t)))$, the index set $\bar{J}$, programs eligible for a status change is defined as

$$\bar{J} = \{j | [\sum_{i=1}^t m_i^t(z(t)) - C_j > 0 \text{ and } z_j(t) = 0] \text{ or } \sum_{i=1}^t [m_i^t(z(t)) - C_j < 0 \text{ and } z_j(t) = 1] \}.$$ 

If $\bar{J} = \emptyset$, the process terminates. Otherwise choose $j \in \bar{J}$ such that $|\sum_{i=1}^t m_i^t(z(t)) - C_j| \geq |\sum_{i=1}^t m_i^t(z(t)) - C_k|$ for all $k \in \bar{J}$, and set $z_j(t+1) = 1 - z_j(t)$ and $z_k(t+1) = z_k(t)$ for $k = 1, \ldots, P, k \neq j$.

Given the selection of $z_j(t+1), j = 1, \ldots, P$, the center computes the allocation of the private good, $z_i(t+1), i = P+1, \ldots, P+S$, as

$$z_i(t+1) - z_i(t) = -\sum_{j=1}^P (z_j(t+1) - z_j(t)) m_i^t(z(t))$$

$$+ \sum_{j=1}^S \sum_{i=1}^P (z(t) - C_j) (z_j(t+1) - z_j(t))$$

where $d_i^t > 0, \sum_{i=1}^S d_i^t = 1$ for $j = 1, \ldots, P$.

In other words, at each iteration the center cannot change the status of a program which either is currently being produced and had a positive surplus ($\sum_{i=1}^t m_i^t - C_j > 0$) or is not being produced and has a negative surplus ($\sum_{i=1}^t m_i^t - C_j < 0$). From the remaining list of programs, the center changes the status of the program with the biggest absolute surplus. For this change, the center charges each station its reported willingness to pay and redistributes any surplus according to fixed sharing rules $d^t_i$.

We could assume that each station is correctly revealing their true preferences and proceed to show that this process is monotonic in that for each social state, each station is at least as well off as with the social state of the previous iteration. Further, under this assumption, the rule has a stable point which is optimal. The difficulty with this lies with the assumption of correct revelation. Indeed, even if stations exhibit minimax behavior, such will not be the case. In the continuous analog of this model in which any non-negative amount of the public good may be produced, Drèze has shown that if the amount supplied is zero, then there are no penalties associated with the under-reporting of preferences since no further decrease in the amount supplied can be contemplated. More generally stated one can prove that truthful revelation is minimax except at corner solutions and, unfortunately, in our problem every feasible allocation is a corner. Thus we are led to search for another alternative.

**IV. AN INDIVIDUALLY INCENTIVE COMPATIBLE PROCEDURE**

Recently a body of literature has evolved in which attention has been devoted to designing mechanisms which elicit the true tastes of the population. Groves [8] has studied one such process in which it is in every individual's interest to announce his true preference for a public good independent of the announcements of the rest of the population. As opposed to the minimax assumption of the previous section, this is the competitive assumption since each member of the
population takes the messages of all other members as given. In environments with separable utility functions, these mechanisms have the property that truthful revelation of preferences is a dominant strategy for each agent and a Pareto optimal decision is taken. In a number of papers, Green and Laffont [5,6,7] have extensively studied this mechanism and have shown that any mechanism which makes Pareto optimal decisions and for which dominant strategies exist is isomorphic to a Groves mechanism.

The difficulty that arises in applying this procedure is that with the exception of [6] none of these papers imposed a constraint on the taxes which may be imposed on each agent and thus bankruptcy was generally feasible. Green and Laffont narrowed the class of Groves' mechanisms to take account of this no-bankruptcy condition in the case of one discrete public good.

The result which we wish to pursue here is to attempt to extend this analysis to the cases of more than one public good and impose a no-bankruptcy condition on each individual and a no-deficit condition on the center. To do this we will need to use the following notation repeatedly:

- $m_i$: The message communicated by station $i$,
- $(S-1)$-tuple of messages communicated by all stations except station $i$,
- $M^i(\mathbf{C}_1,\ldots,\mathbf{C}_p,\mathbf{b}_1,\ldots,\mathbf{b}_r,\mathbf{m})$: The message space correspondence of station $i$,
- $\Omega = \{0,1\}^P \times \mathbb{R}_+^S$: The set of possible allocations for which no station faces bankruptcy,
- $\Omega = \{(x,y) \in \Omega \mid \sum_{k=1}^{P} C_k x_k + \sum_{k=1}^{S} y_k \leq \sum_{k=1}^{S} b_k\}$: the set of feasible allocations for which no station faces bankruptcy and for which the center does not incur a deficit.

With these definitions, we have imposed a certain structure on the institution. The set of admissible messages may depend only on observable data (namely, the costs of the programs, the budgets of the stations, and the messages of all other stations). Secondly, the set of possible allocations, $\Omega$, has restricted allocations of the private good to the non-negative orthant to ensure no bankruptcy can occur. Finally, the set of feasible allocations, $\hat{\Omega}$, guarantee that the center cannot run a deficit since, if we regard $t_k = b_k - y_k$ as the lump sum tax paid by station $k$, the set $\Omega$ requires

$$\sum_{k=1}^{S} t_k \geq \sum_{k=1}^{P} C_k x_k.$$ 

Given the message space, we may now define our mechanism as a mapping of messages into the set of feasible allocations. More specifically let $F: \mathbb{M}^i \rightarrow \hat{\Omega}$ where $\mathbb{M}^i$ denotes the range of $M^i$.

In order to compare different allocations in $\hat{\Omega}$, we will need to assume that each station has a preference relation, $R^i$, which is defined on $\hat{\Omega}$. This relation is assumed to be complete, transitive, and reflexive. Further, let $P^i$ denote the asymmetric part of $R^i$ and let $I^i$ denote the symmetric part.
With these definitions we can now define the notion of admissible dominant strategies:

**Definition:** F is non-strategic if and only if

\[ \forall i = 1, \ldots, S, \forall m, m' \in M_i \text{ such that for each } m, m' \in M_i, \]

\[ F(m_i, m'_i) = \begin{cases} 1 & \text{if } m'_i = i(\hat{m}_i) \\ 0 & \text{otherwise} \end{cases} \]

\[ F(m_i, m'_i) = \begin{cases} 1 & \text{if } m'_i = i(\hat{m}_i) \\ 0 & \text{otherwise} \end{cases} \]

for all \( \hat{m}_i \in M_i. \)

In the case where the dominant strategy for each agent was his true preference, the mechanism is said to be strongly individually incentive compatible (SIIC). Such may not be the case here, in as much as the SIIC strategy may be inadmissible (as in [6]).

The definition of Pareto optimality creates some difficulty in this setting. The ordinary definition of optimality requires the mechanism, F, to exactly balance the budget (i.e.,

\[ \sum_{j=1}^{P} t_j - \sum_{i=1}^{S} C_i x_i = 0 \]

Green and Laffont [7], this requires that F can not be non-strategic. Due to this, we have chosen to relax the usual efficiency requirement with the following:

**Definition:** \((x, y)\) is said to Pareto-dominate \((x', y')\) if and only if

(i) \(x_k \neq x'_k\) for some \(k = 1, \ldots, P\)

(ii) \((x, y)_j \geq (x', y')_j\) \(\forall j = 1, \ldots, S\) and \((x, y)_j \geq (x', y')_j\) \(\forall j = 1, \ldots, S\).

and

(iii) \(x_k \leq y_k\) \(\forall k \in S\)

Thus we will consider one alternative to Pareto-dominate another if and only if it differs on the set of programs selected, it is non-distributive and the usual definition of Pareto domination, (ii), holds. We will call this mechanism weakly efficient when \(F(m)\) selects the maximal elements of \(\hat{m}\).

With these definitions, we hoped to construct a class of mechanisms which were weakly efficient and non-strategic. Indeed, in [6], the authors have constructed such a class in the case of a single public good. What we discovered in the case \(P \geq 2\), was that no such mechanism existed. This can be shown by way of the following example. Let us begin by examining a special class of preferences which we denote by \(L\). A preference relation is in \(L\) if there is a vector \((v_1, \ldots, v_p)\) of non-negative numbers such that for each \((x, y)\), \((x', y') \in \Omega\), \((x, y)_j \geq (x', y')_j\) \(\forall j = 1, \ldots, S\) if \(x_1 < y_1\). Now suppose there are two stations with preference relations in \(L\) and two programs with \(C = C_1 = C_2 = B^1 + B^2\). If station \(i\) has preferences \((v^i_1, v^i_2)\) such that

\[ v^1_1 + v^1_2 > C_1 \text{ and } v^2_1 + v^2_2 < C_2 \]

where

\[ C_1 > v^1_1 > v^1_1 + B^1 \text{ and } C_1 > v^2_1 > B^2, \]

weak efficiency requires that \(F(m) = (1, 0, 0, 0)\). Alternatively, if stations have preferences \((v^i_1, v^i_2)\) such that

\[ v^1_1 + v^2_1 < C_1 \text{ and } v^1_2 + v^2_2 < C_2. \]
where 
\[ c_2 > \frac{v^2}{v_1} > \frac{v^2_1}{v_2} > b^2 > b_1 \] and 
\[ c_1 > \frac{v^{-1}}{v_2} > b^1, \]

weak-efficiency requires that the second program is produced, i.e.
\[ F(m) = (0,1,0,0). \]

Now let us suppose that station 1 has preferences given by \( (v^1_1, v^2_1) \) and station 2 has preferences \( (v^2_1, v^2_2) \). Thus 
\[ v^1_1 + v^2_1 > c_1 \] and 
\[ v^1_2 + v^2_2 > c_2 \]
where 
\[ v^1_2 > v^1_1 \] and 
\[ v^2_1 > v^2_2. \]

Here, weak-efficiency only requires that either program be produced. However, if \( F(m) = (0,1,0,0) \), then station 2 has the incentive to misreveal its preferences as \( (v^2_1, v^2_2) \) which will cause the first program to be produced. On the other hand, if \( F(m) = (1,0,0,0) \), then station 1 will misreveal his preferences as \( (v^1_1, v^1_2) \) to guarantee production of program two. Thus this example proves the following impossibility result:

**Proposition:** If \( P \geq 2 \) and \( F \) is weakly efficient then \( F \) is not nonstrategic.

V. Concluding Remarks

From our search for a satisfactory mechanism we are left with the following observation. If we wish to continue with the class of deterministic mechanisms we must allow for some strategic behavior, but this creates a major implementation difficulty.

The mechanisms described in the previous section are static mappings between agents' messages and final allocations. This static property is relatively unobjectionable when the agents have dominant strategies but if dominant strategies do not exist, such mechanisms must be implemented as adjustment procedures.

With the exception of Smith [11], there has been little attention paid to the problem of modeling such institutions and the theoretical analysis of the associated adjustment procedures is lacking. Due to this we are left to analyze the properties of competing institutions through the use of laboratory experiments (see [4]) in order to compare their properties under different evaluative criteria.


