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THE GREAT FISH WAR: A COOPERATIVE SOLUTION

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ABSTRACT

The competitive allocation of a common property resource is analyzed taking explicit account of the fact that the resource users must confront each other repeatedly. This means that future retaliation for noncooperative behavior is possible. The likelihood of enforcing cooperative behavior with the credible threat of retaliation is analyzed using the theory of repeated games.
THE GREAT FISH WAR : A COOPERATIVE SOLUTION

Introduction

In a recent paper Levhari and Mirman (1980) present an elegant and very thought provoking analysis of the competitive use of a commonly owned renewable natural resource. (Their leading example is the exploitation of an international ocean fishery.) They rule out the possibility that fishermen explicitly cooperate with one another in harvesting the resource because of the excessive cost of enforcing agreements. Instead they examine the time path of resource consumption derived under the assumption of Cournot Nash behavior in which each fisherman selects a rate of harvest function, contingent on the size of the stock, to maximize his present value flow of utility. Furthermore, the harvesting strategies are required to be perfect in the sense of Selton (1975). (We discuss the concept of perfectness in more detail below.) The model that is derived from this analysis performs nicely in that it yields predictions which are consistent with what we typically observe with the management of common property resources - the resource is squandered as competing users vie with each other to obtain a larger share of the stock.

It does seem, though, that we do occasionally observe individuals efficiently utilizing common property without the aid of a regulatory body to oversee their activities. In these situations an important factor influencing everyone's behavior is that independent resource users must interact with one another on a continual basis. This suggests that users could construct cooperative schemes for observing the resource that are self policing. The incentives for one user to deviate from the cooperative arrangement would be eliminated by the threat of retaliation by others. In the context of the Levhari and Mirman fishing example, such schemes would presuppose a dynamic equilibrium in which each fisherman (or firm) would monitor the behavior of its rivals over time. A firm's harvesting decision would depend on the current stock as well as on the previous behavior of its competitors. Each fisherman would agree to harvest at an efficient rate contingent on observing the same behavior from his rivals. Assuming that firms could accurately observe the total catch in each period, one fisherman's deviation from the cooperative agreement would be recognized by his rivals who would retaliate by collectively adopting the noncooperative behavior as described by Levhari and Mirman.

The purpose of this note is to investigate the model we have described above for its predictions on the sustainability of cooperative resource use, in which agreements are enforced by threat of retaliation. We use the Levhari-Mirman example to relate the prospects for maintaining cooperative arrangements to (a) the number of users and their ability to monitor each other, (b) the time rate of discount, and (c) the growth potential of the resource.
II. THE MODEL

The formal apparatus for our analysis is the theory of repeated games. We envision the interaction between fishermen as a repetition of single period game where independent users compete with each other for the consumption of a commonly owned resource. Following Levhari and Miriman let $X_t$ be stock of fish at time $t$, which grows, if left alone, according to the equation $X_{t+1} = X_t^\alpha$ for $\alpha \in (0,1)$. Assuming the stock does not exceed its maximum sustainable size, which is 1, the parameter $\alpha$ is a measure of the natural growth potential of the resource. For $\alpha$ close to 1, for example, the stock grows very sluggishly, and it is practically nonrenewable like a pool of oil.

Assume there are $N$ identical and independent users of the resource. Denote $c^i_t$ as the consumption of user $i$ at time $t$, and let $u(c^i_t) = \log c^i_t$ be individual $i$'s utility derived from consumption.

It is easy to derive the cooperative - joint maximizing program for utilizing the stock. Due to the concavity of $u(c^i_t)$ the optimal program involves equal consumption by all users at all times, so that $c^i_t = c_t$ for $i = 1, \ldots, N$. Let $V^*_t(X_t)$ denote the value of the discounted stream of utilities for each user given stock $X_t$ remains with $t$ periods left in the planning horizon. The value functions in succeeding periods must satisfy this recursion relationship

$$V^*_t(X_t) = \max_{c_t} \left[ \log c_t + \beta V^*_{t-1} \left( (X_t - Nc_t)^\alpha \right) \right]$$

in which the value $X_t - Nc_t$ is the remaining stock of fish at the end period $t$ which becomes $(X_t - Nc_t)^\alpha$ in the following period. $\beta$ is the common discount factor. If one extends the planning horizon to an infinite number of periods then the stationary consumption rule and value function are given respectively by

$$c^*(X) = \frac{(1-\alpha)X}{N} \quad (2a)$$

$$V^*(X) = \frac{1}{1-\beta} \log(X) + \frac{\log \left( \frac{(1-\alpha)X}{N} \right)^{\alpha/(1-\alpha)}}{1-\beta} - \frac{\log(\beta)}{1-\alpha} \quad (2b)$$

These expressions are derived in the Appendix.

As long as all users adopt the cooperative mode of fishing, they each follow the harvesting strategy $c^*(X)$ given by (2a) and receive utility $U(c^*(X))$. If at some time one of the firms catches more than the amount, $c^*(X)$, it is entitled to, then we assume that the cooperative agreement breaks down, and in the next period all fishermen revert back to permanently pursuing their noncooperative Nash Cournot strategies. Levhari and Miriman derived the stationary consumption strategies and value functions for the symmetric two person Cournot Nash equilibrium. The corresponding functions for the $N$ person equilibrium are derived in the appendix, and they are given by

$$c^0(X) = \frac{(1-\alpha)X}{N-(N-1)\alpha} \quad (3a)$$
\[ V^0(x) = \frac{1}{1-\alpha^\beta} \log x \]
\[ + \frac{1}{1-\beta} \left\{ \log \left( \frac{1-\alpha^\beta}{N-(N-1)\alpha^\beta} \right) \log \left( \frac{\alpha^\beta}{N-(N-1)\alpha^\beta} \right)^{\alpha/(1-\alpha)} \right\} \]
\[ - \frac{1}{1-\alpha^\beta} \log \left( \frac{\alpha^\beta}{N-(N-1)\alpha^\beta} \right)^{\alpha/(1-\alpha)} \]

in which variables with superscript \(\alpha\) denote noncooperative equilibrium quantities.

Comparing equations (2) with equations (3) we see that they coincide only when \(N=1\); otherwise when \(N>2\), \(c^0(x) > c^*(x)\) so that the resource is depleted too rapidly when firms don't cooperate. One can also show that \(V^0(x) < V^*(x)\). Hence the breakup of the cooperative caused by the deviation of a single firm, (hereafter called the cheater), would harm all users. Most important however is that the cheater would suffer after the breakup and this is intended to be a deterrent to his cheating.

Define a deterrent strategy to be the rule that a firm adopts the cooperative harvesting strategy \(c^*(x)\) for all periods until it detects the total harvest exceeding the cooperative each \(Nc^*(x)\) whereupon it permanently follows the noncooperative catch rule \(c^0(x)\) for the remainder of time. A cheat strategy is simply defined as a deviation from the cooperative harvesting rule by a firm at some time. It is possible to sustain a cooperative solution as a Nash equilibrium whenever the net returns to a single firm from cheating are negative given that all other firms adopt deterrent strategies. Let \(V^d(x)\) be the net value to a firm from cheating on the cooperative agreement given the current stock is \(x\), and that all other firms are utilizing deterrent strategies.

III. COOPERATIVE AND NONCOOPERATIVE SOLUTIONS

One of the first and probably the most important application of repeated games to economics was put forth by Stigler (1964) who suggested that oligopolistic firms would maintain a monopolistic price by threatening price chisellers with punitive actions. This idea was later formalized by Friedman (1971), and numerous authors have written on repeated oligopoly and strategic market games since then. Our model of repetitive play between resource users differs from the standard market games in a couple of respects. A central feature of our analysis is that previous actions of the users affect the current stock and thereby they determine the present production possibilities for the firms. With the exception of a few models that allow for inventories, repeated market games assume that the history of play does not affect the rules of the game in succeeding periods. Another distinguishing feature of our analysis is that the duration of our game is endogenously determined by the users in how they affect the availability of the resource. For our example it turns out that the resource is never exhausted, and hence the game goes on forever. In most market games that are analyzed there is no natural way to specify the number of periods of play. This is troublesome because infinite repeated games (supergames) sometimes contain cooperative equilibria which can not be sustained in finite repeated games. A final important aspect of our model is that unlike most repeated market games, the cooperative and noncooperative strategies that are used in single plays of the game depend on the discount rate, (see equations 2 and 3). To identify the role of some of these features
It is define by

\[ V^d(x) = \max_{c^d} \left[ \log(c^d) + \beta V^0((x-(N-1)c^*) - c^d) \right] - V^*(x) \tag{4} \]

The expression in square brackets in (4) represents the immediate utility from cheating plus the discounted stream of utility that follows once the deviation is detected and all users adopt noncooperative behaviour. The last term in (4) is the utility that the cheater foregoes by foreseen the cooperative agreement. A Nash equilibrium in deterrence strategies requires that \( V^d(x) \) be negative.

Furthermore, if \( V^d(x) \) is negative then the Nash equilibrium in deterrent strategies also satisfies the perfectness property of Setten (1975). Roughly, perfectness requires that each firm's strategy must be an optimal response to the behaviour of its rivals under all conceivable histories of play and, not just those histories that occur in equilibrium. Perfect strategies do not admit "incredible threats" in which firms enforce cooperative play by threatening reprisals that they would never carry out if given the opportunity. The deterrence strategies we have introduced above are credible. In particular if a firm ever observes a total harvest that exceeds the allowable cooperative catch, \( Nc^*(x) \), at time \( t^0 \), its best response is to pursue the noncooperative strategy \( c^0(x^*) \) for all \( t > t^0 \). This is because all other firms will have observed the deviation and they will also adopt the noncooperative harvesting rule. On the other hand, if a firm has not observed a deviation prior to time \( t^0 \), then it is best for it to continue cooperating provided that \( V^d(x) < 0 \). Hence the deterrence strategy is in fact a best response strategy for all conceivable histories of play that the firm may encounter.

in our analysis we now consider how the incentives to cooperate are affected by variations in the parameters of our model parameters.

Variations in the Discount Factor

Independent firms may enforce cooperative management of the resource whenever the return from cheating, \( V^d(x) \), is negative. Substituting for \( V^0 \) and \( V^* \) from equations 2 and 3 respectively into equation 4, one can show that the optimal cheat, \( c^d(x) \), is given by

\[ c^d(x) = \frac{(1-\alpha)(Nn \alpha - \alpha \beta + 1)}{N} \tag{5} \]

According to equations (2.a) and (5) \( c^d(x) > c^*(x) \) whenever \( \beta > 0 \) and \( N > 2 \). Cheating involves consuming more than one's share of the cooperative harvest. Notice however that \( c^d(x) > c^*(x) > \frac{x}{N} \) as \( \beta \rightarrow 0 \). This suggests that the incentives to cheat may be reduced if the users collectively agree to consume the resource very rapidly.

Substituting from equations (2b), (3b) and (5) into equation (4) one obtains a complicated expression for \( V^d(x) \) in terms of \( \beta, N, \) and \( \alpha \) which is most easily analyzed by computing the value of \( V^d \) for different combinations of values for our parameters. This was done for various values of \( \beta \) and \( \alpha \) ranging between 0 and 1 and for values of \( N \) ranging between 2 and 100. (A complete listing of the calculations is available from the authors).

We find that the effects of variations in \( \beta \) on the incentives for firms to cheat conform to the results normally reported in the repeated games literature. Cooperative behaviour can not be enforced when the discount factor becomes small. The calculations appearing in Table 1 are typical of the results we obtain. (Notice only the sign of \( V^d(x) \).)
equal to the difference in the values of the cooperative and non-cooperative programs of resource use. This loss which is a deterrent to cheating increases with N because the rents earned under noncooperative exploitation of the resource decline very rapidly as the number of users grows. This decline in rents (a phenomena which has been dubbed "the Tragedy of the Commons" by Hardin (1968)) becomes a greater deterrent to defection so that the resource is harvested cooperatively when N is sufficiently large.

The conventional view that cooperative arrangements among members of a cartel, between oligopolistic firms, or among independent resource users are likely to breakdown with large numbers rests on several arguments. One of them is that reaching agreement on a common plan is more difficult with large numbers because of a greater degree of diversity in preferences and because of the greater costs of negotiation. Another argument is that it becomes increasingly difficult to identify and hence to punish a defecting firm when the number of producers grows. Our model abstracts from these factors, by assuming that all users are identical, and that defections from the cooperative can be detected costlessly. Hence our result is not to be interpreted as a counter example to the conventional wisdom on the stability of cooperatives with large numbers. Rather it suggests that the cooperative's ability to punish a defector by having the members break up and behave competively may increase with its size. This is a way in which size adds to the stability of the cooperative and it must be balanced off against the increased difficulty of reaching agreements and monitoring behaviour which also results with larger numbers.

Variations in the Growth Rate

The ability of the stock to reproduce itself increases as \( \alpha \) ranges from 1 to 0. An example of the effects of variations in \( \alpha \) on \( V^d(X) \) appears in Table 3. One can show that as \( \alpha \to 0 \), that \( |V^d(X)| \to 0 \). Also we note from equations (2.a) and (5) that \( c^d(x) \to c^*(x) \to X/N \) as \( \alpha \to 0 \). In this case the resource reproduces itself so rapidly that the cooperative agreement calls for consuming virtually all the available resource in each period. Under these circumstances it is unimportant whether the resource is exploited in a cooperative or competitive manner.

As \( \alpha \) increases, the incentives to cheat decline, and it may be possible to enforce a cooperate agreement using deterrent strategies as in Table 3. Notice that the ability of the resource to grow diminishes as \( \alpha \) increases. As \( \alpha \to 1 \) the resources becomes non-renewable. Our results suggest that cooperative management of the stock will be most likely to occur when the resource is scarce in that it has limited capacity to grow.
and the relative changes in it for variations in our parameter are meaningful). Cheating is deterred for \( \beta \) sufficiently large, the incentives to cheat increase over a range as \( \beta \) declines, and they go to zero as \( \beta \) approaches zero. The last result confirms our suspicion that cheating may not be a significant problem when users have a high rate of discount.

Finally we note that in all of our results the tendency for firms to cooperate or cheat is completely independent of the current size of the resource. This is because terms involving \( X \) cancel each other out in the expression for \( V^d(X) \). Hence initial conditions are unimportant in shaping the equilibrium behaviour of the resource users. This property is almost certainly peculiar to our example and we would not expect it to hold in general.

Variations in the Number of Users

The viability of sustaining cooperative management of the resource would seem to depend critically on the number of independent firms who harvest it. The conventional wisdom among most resource economists on this issue is aptly put forth by Colin Clark (1980, pp. 131) who states,

"With the possible exception of a few fisheries exploited exclusively by the "factory" fleets of the USSR and other Eastern bloc countries, individual producers will normally be too numerous to allow a cooperative scheme to operate stably in practice."

To investigate this claim in the context of our example we differentiate \( V^d(X) \) with respect to \( N \), and obtain the following expression

\[
\frac{dV^d(X)}{dN} = \left\{ -\frac{1}{N} - \frac{\alpha \beta}{N \alpha \beta + 1 - \alpha \beta} \right\} \left( \frac{1}{1 - \alpha \beta} \right) - \frac{\beta (1 - \alpha \beta)}{N (1 - \alpha \beta) + \alpha \beta} \left[ \frac{1}{1 - \beta} - \frac{\alpha}{1 - \alpha \beta} \right] \frac{1}{1 - \alpha} + \frac{1}{(1 - \beta)N}
\]

(6)

Now suppose \( \beta \to 1 \), then equation (6) implies that all but the terms involving \( 1/(1-\beta) \) vanish so that we have

\[
\lim_{\beta \to 1} \left\{ \text{sign} \left( \frac{dV^d(X)}{dN} \right) \right\} = \text{sign} \left\{ \frac{1}{N - n(1-\alpha) + \alpha} \right\}
\]

(7)

\[
= \text{sign} \left\{ \frac{N(1-\alpha) + \alpha - N}{N(1-\alpha) + \alpha - N} \right\}
\]

\[
= \text{sign} \left\{ \alpha(N) \right\} < 0
\]

Recall that (according to the previous section) it is possible to maintain cooperative harvesting for a given number of users when \( \beta \) is sufficiently large. Equation 7 suggests that under these circumstances that the incentives for independent users to cooperatively exploit the resources are enhanced as the number of firms increases. This finding shows up in all our numerical results for plausible values of \( \beta \geq 0.8 \) which correspond to rates of discount equal to or less than 25%.

An example of this result is represented in Table 2.

The intuitive explanation for this striking result is that when \( \beta \) is sufficiently large a perspective cheater must pay attention to the future utility loss he will suffer because of the adverse reaction by his rivals to his defection. After the cheat is discovered, the defector (as well as all other firms) suffers a loss in utility
An interesting feature of our model is that it is possible for firms to collectively internalize the costs that they impose on one another and to produce efficiently without the aid of outside regulation. The model can be viewed as being a formalization of the Coasian (1960) solution to externality problems in a dynamic context. This suggests to us a different approach to regulation from that which is normally espoused. According to our scheme, the regulator would try to create environments which were conducive to allowing the firms to coordinate and police their own behaviour. Conceivably this would involve the regulation in controlling the number of users, in facilitating communication between them, and providing information about their activities to be used by the firms to monitor each other. This would be in contrast to using externally administered taxes and subsidies, or direct controls to achieve efficiency.

The requirement of perfect monitoring is too strong to be satisfied in practice, particularly in a stochastic environment. It may not be possible to enforce perfectly cooperative behaviour using deterrent strategies with imperfect detection. However it may be feasible to achieve somewhat less efficient (or second best) cooperative outcomes. An interesting topic for future research would be to identify minimal conditions under which the use of deterrent strategies of some form can improve the equilibrium allocation of resources in repeated games.
Table 1
The Effect of an Increase in the Discount Factor
$\beta$ on $V^d$, (with $N=10$, $\alpha=.5$)

<table>
<thead>
<tr>
<th>Discount Factor ($\beta$)</th>
<th>$V^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.26185</td>
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<tr>
<td>.3</td>
<td>.49783</td>
</tr>
<tr>
<td>.5</td>
<td>.37534</td>
</tr>
<tr>
<td>.7</td>
<td>-.38532</td>
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<tr>
<td>.8</td>
<td>-2.15661</td>
</tr>
<tr>
<td>.9</td>
<td>-7.39909</td>
</tr>
</tbody>
</table>

Table 2
The Effect of an Increase in the Number of Users, $N$
on $V^d$, (with $\beta=.9$ and $\alpha = .5$)

<table>
<thead>
<tr>
<th>Number of Users, $(N)$</th>
<th>$V^d$</th>
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<tbody>
<tr>
<td>2</td>
<td>-.82467</td>
</tr>
<tr>
<td>3</td>
<td>-1.98514</td>
</tr>
<tr>
<td>4</td>
<td>-3.05145</td>
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<td>5</td>
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<tr>
<td>6</td>
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</tr>
<tr>
<td>100</td>
<td>-21.08593</td>
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</table>

Table 3
The Effect of Increases in the Growth Potential,$\alpha$, on $V^d$ (with $N=10$, $\beta=7$)

<table>
<thead>
<tr>
<th>Growth Potential ($\alpha$)</th>
<th>$V^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.11091</td>
</tr>
<tr>
<td>.3</td>
<td>-.07816</td>
</tr>
<tr>
<td>.5</td>
<td>-.58532</td>
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<tr>
<td>.7</td>
<td>-1.40306</td>
</tr>
<tr>
<td>.9</td>
<td>-2.66153</td>
</tr>
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</table>
APPENDIX

a) Derivation of the Cooperative Solution

Our derivation is adopted from Levhari and Mirman (1980 pp. 329 footnote 10). We suppose there are N identical users (Levhari and Mirman assume N=2). The cooperative program is derived as the solution to

\[
\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t N \log \left( c_t \right)
\]

subject to \(X_{t+1} = (X_t - Nc_t)^{a} \). The optimal policy must satisfy the functional equation, which for this case is written

\[
\frac{1}{c_t} = \frac{\alpha \beta}{c_{t+1}} X_t^{a-1}
\]

(We are proceeding nonrigorously here, see Levhari and Mirman, pp. 328 footnote 9). Assuming the stationary solution is of the form \(c^* = \lambda^* X\), and substituting for \(c\) in (A2) we obtain

\[
\frac{1}{\lambda^* X} = \frac{\alpha \beta}{\lambda^* (\lambda - \alpha N^* X)}
\]

or

\[
\lambda^* = \frac{1 - \alpha \beta}{N}
\]

One can show by induction that the value function \(V^*(X)\) is of the form \(V^*(X) = m \log (X) + \overline{m} \). \(V^*(X)\) must satisfy the recursion equation

\[
V^*(X) = \max_{c} \left[ \log (c) + \alpha \beta \ m \log (X-Nc) + \beta \overline{m} \right]
\]

in which \(m\) and \(\overline{m}\) are constants. Differentiating (A4) with respect to \(c\) implies that \(c^*\) must satisfy

\[
\frac{1}{c^*} \frac{\partial}{\partial X} (X-N c^*) = 0
\]

Substituting for \(c^* = (1-\alpha \beta) X/N\) in (A5) implies

\[
m = \frac{1}{1-\alpha \beta}
\]

To solve for \(\overline{m}\) we note that \(X^*\) the steady state stock equals \(\alpha \beta \alpha/(1-\alpha)\) (see Levhari and Mirman pp. 329 footnote 11). Hence we know

\[
V^*(X^*) = m \log (X^*) + \overline{m} = \frac{1}{1-\beta} \log \left( \frac{(1-\alpha \beta) (\alpha \beta)^{\alpha/(1-\alpha)}}{N} \right)
\]

so that

\[
\overline{m} = \log \left( \frac{(1-\alpha \beta) (\alpha \beta)^{\alpha/(1-\alpha)}}{N} \right) - \log \left( \frac{(\alpha \beta)^{\alpha/(1-\alpha)}}{1-\alpha \beta} \right)
\]

b) Derivation of the Noncooperative Solution

Adopting the derivation in Levhari and Mirman (pp. 327-8 footnote 9) to our N user case, one can show by induction that the equilibrium consumption strategies are of the form \(c^0 = \lambda^0 X\) and that \(\lambda^0\) must satisfy the functional equation
\[
\frac{1}{\lambda^o X} = \frac{\beta_\lambda (1-(N-1)\lambda^o)}{\lambda^o (X-N\lambda^o X)}
\]

(B1)

Using (B1) to solve for \(\lambda^o\) yields

\[
c^o = \lambda^o X = \frac{(1-\alpha\beta)X}{N-(N-1)\alpha\beta}
\]

(B2)

One can show by induction that the value function \(V^o_i(X)\), for user \(i\) is of the form

\[
V^o_i(X) = K \log X + \bar{K}
\]

(B3)

\(V^o_i(K)\) must satisfy the recursion equation.

\[
V^o_i(X) = \max \left[ \log c_i + \alpha\beta K \log \left( \frac{X-c_i}{\sum_{j \neq i} c_j} \right) + \beta \bar{K} \right]
\]

(B4)

Differentiating (B4) with respect to \(c_i\), and recognizing that \(c_i = c_j = \lambda^o X\) for all \(i\) and \(j\) we obtain

\[
\frac{1}{c^o} - \frac{\alpha\beta k}{X-Nc} = 0 \quad \text{or} \quad \frac{1}{\lambda^o} - \frac{\alpha\beta \bar{K}}{1-N\lambda^o} = 0
\]

(B5)

Combining (B2) and (B5) one can show

\[
K = \frac{1}{1-\alpha\beta}
\]

(B6)

The steady state value for the stock \(\lambda^o\), must satisfy

\[
\lambda^o = (X^o - \frac{N(1-\alpha\beta)\lambda^o}{N-(N-1)\alpha\beta})^\alpha
\]

(B7)

which implies

\[
\lambda^o = \left( \frac{\alpha\beta}{N-(N-1)\alpha\beta} \right)^{\alpha/(1-\alpha)}
\]

(B8)

In order to solve for \(\bar{K}\) we note

\[
V^o(X^o) = K \log X^o + \bar{K} = \frac{\log(\lambda^o X^o)}{1-\beta}
\]

(B9)

implying that

\[
\bar{K} = \frac{1}{1-\beta} \log \left( \frac{(1-\alpha\beta)}{N-(N-1)\alpha\beta} \left( \frac{\alpha\beta}{N-(N-1)\alpha\beta} \right)^{\alpha/(1-\alpha)} \right)
\]

(B10)
FOOTNOTES

1. Clark (1980) also contains an analysis of the exploitation of an open access fishery in a model that differs significantly from that used by Levhari and Miran.

2. In contrast to this, Munro (1978) investigates a case where independent fishermen explicitly coordinate their actions to maximize the present value flow of rents from the fishery. The theory of cooperative games is employed to determine the distribution of rents among various users.

3. Roughly, a perfect equilibrium requires that each firm's strategy be a best response in all conceivable situations. Such strategies are derived using a dynamic programming approach as in Levhari and Miran (1980). Perfectness rules out the possibility of incredible threats, whereby firms try to manipulate each other with threats which they would never execute. Reingaum and Stokey (1981) contains a particular cogent discussion of perfectness in the context of models of natural resource exploitation. Eswaran and Lewis (1982a,b) and Reingaum and Stokey discuss the properties of perfect equilibria in models of common property resources.

4. See Hardin (1968) for a graphic account of the general mismanagement of open access resources.

5. The communal management of common water supplies, grazing land, wilderness areas, and road systems in rural areas are examples of open access resources that are rationed and controlled by informal agreements. There are also certain small coastal fisheries in North America and Europe which are efficiently managed by loose cooperatives or alliances of fishermen. Private communication Peter Pearse

6. The standard reference for the theory of repeated games is Friedman (1977)

7. It is only necessary for firms to monitor the total catch, rather than the individual catch of each user to detect if one of them has broken the agreement.

8. For example see the papers by Green (1980) and Radner (1980a,b).


10. One can show that \( \lim_{\beta \to 0} V_d(X) = 0. \)

11. We note that when \( \beta \) is small, increases in \( N \) seem to increase the incentives to cheat.

12. A good discussion of this view is contained in Scherer (1980, Chpt.7).


BIBLIOGRAPHY


