STRATEGIC COMMITMENT UNDER UNCERTAINTY
WITH PRIVATE INFORMATION

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ABSTRACT

The paper takes the approach that observed differences in firm size and market share may be explained by producers having access to different information at the time of their investment decisions. This view is examined in a duopoly production model where each firm commits itself to a production plan based on proprietary information on future market conditions. The model is used to analyze the effect of different information structures on market conduct and performance.
I. Introduction

The "New I.O." is concerned with explaining how markets evolve under various types of strategic interaction between independent firms. Schelling's (1960) notion of precommitment is central to this development. The idea is that a firm which makes irreversible investments in plant capacity, for example, commits itself to large scale production later on. In taking this action first, the firm guarantees a big share of the market for itself, and it may even discourage competition from other firms in the future. Besides explaining investment in capacity, the first mover paradigm has also been employed to analyze advertising, product selection, choice of location, and preemptive patenting and innovation as strategic market activities. The recent review articles by Dixit (1982), Encausa (1982), Salop (1978) and Schmalensee (1981) nicely describe the various models of strategic commitment.

These models have been very helpful in forming the modern theory of industrial structure and in the application of antitrust law to shaping industrial structure. However, the first mover paradigm as it is normally presented in the literature is incomplete in that it does not specify how the first mover is selected. One would like a model which endogenizes the order of play or a model that explains how the ex post positioning of players in strategy space evolves. One method adopted by Berman and Schötter (1981) is to have the order of play determined as part of the super game equilibrium (assuming repetitious play).

An alternative approach adopted in this paper is to assume that all firms may make strategic decisions at the same time, but that they are imperfectly informed about market demand when making their decisions. Jovanovic (1981) makes a similar assumption in his model of location choice. Specifically, in our model, firms decide on plant capacity in period one before market conditions are known. Producers make their decisions on the basis of private information they receive in the form of a demand signal. In period two, firms choose a level of production contingent on the state of demand and their plant size. Our analysis suggests that in making investment decisions some producers will elect to play it safe, while others will charge ahead and commit themselves to establishing a dominant market position. This scenario makes sense if firms either have disparate information about the payoffs they will receive from the ensuing market game, or they process the same information in different ways. Players having an optimistic view of the future will want to commit themselves to large scale production by building a big plant. Less optimistic players will be more cautious. Viewing this process, ex post, after the firms have moved, we might attribute a first mover advantage to one of the producers, when in fact, differences in initial beliefs and information would explain our observation.

In our analysis the information environment that firms operate in is all important in determining market structure and performance. The information that firms base their investments on is described by three characteristics (a) a producer's prior knowledge of demand before it receives its demand signal, (b) the precision of its demand signal and (c) the correlation in signals between different firms. One of our concerns in this paper is to understand how exogenous demand uncertainty
affects precommitment behavior, and to predict what degrees of accuracy and overlap in the private information that firms observe produce the largest differences in capacity and market shares, between firms. The other purpose of our study is to assess the private and social value of endowing producers with better information or allowing them to share and to act on common information.

In Section II of the paper we present our model of strategic investment in capacity under imperfect information. For tractability we investigate a duopoly market where producers face an unknown linear demand function with quadratic production costs. The unknown demand intercept is normally distributed and each firm's knowledge of the distribution of demanded is updated based on a private signal that it receives (also normally distributed). In Propositions 2.1 and 2.2 we demonstrate that there exists a perfect Bayesian Nash equilibrium in which producers choose a level of cost-shifting investment contingent on their information.

Section III explores the comparative static properties of symmetric equilibria in which firms receive identically distributed signals. The effects of increasing demand uncertainty on industry structure and performance are summarized in Proposition 3.1. Propositions 3.2, 3.4 and 3.5 deal with the reactions of firms to receiving more accurate demand signals. Proposition 3.3 demonstrates that industry performance improves as the demand signals that firms receive become less positively correlated. The prospects for allowing firms to pool their information are assessed in Propositions 3.6 and 3.7.

Section IV treats the asymmetric case in which firms differ in how knowledgeable they are. The private and social value from having one producer become better informed than another is characterized in Propositions 4.1 and 4.2.

Section V summarizes our findings and concludes with some speculations about expanding our analysis to the cases in which firms can acquire additional information at a cost and they can transmit information to each other.
II DERIVATION OF STRATEGIC COMMITMENT EQUILIBRIUM

IIa Assumptions and Notations

We assume there are two firms in the industry. (Generalization to N firms is discussed in Section V). The equation for market price, \( p \), as a function of the output of firms 1 and 2 denoted by, \( q_1 \) and \( q_2 \) respectively, is

\[
p = \alpha - q_1 - q_2.
\]

The demand intercept, \( \alpha \), is random, and its stochastic specification is discussed below. Revenue for producer \( i \) is given by

\[
(2.1) \quad (\alpha - q_1 - q_2)q_i \quad ; \quad i = 1, 2
\]

Let \( C(q_i, X_i) \) be the cost of production for firm \( i \). It is defined by

\[
(2.2a) \quad C(q_i, X_i) = \gamma q_i + \frac{(X_i - q_i)^2}{2} ; \quad i = 1, 2, \quad \gamma > 0
\]

One can think of \( X_i \) as a measure of "ideal" production capacity. By design, unit production costs are minimized at output equal to \( X_i \). The idea behind (2.1a) is that in period 1 firms commit themselves to a choice of \( X_i \), by contracting for specified amounts of labor services, raw inputs, and capital equipment for use in production in period two. Deviations in production away from \( X_i \) are costly to the firm because of the difficulty of renegotiating contracts for the delivery of capital, labor and raw inputs. The cost of adjusting output will vary across industries. We could multiply the quadratic term in (2.2a) by a positive constant, ranging between zero and infinity to reflect the flexibility of the production process. However, we simply assume the constant is equal to one in what follows.

Rewriting (2.2a) slightly we obtain

\[
(2.2b) \quad C(q_i, X_i) = (\gamma - X_i)q_i + \frac{q_i^2}{2} + \frac{X_i^2}{2} ; \quad i = 1, 2
\]

which suggests another interpretation for our model. We can regard \( X_i \) as an investment in a cost reducing device, where the cost of investment is \( X_i^2/2 \). For convenience we will refer to \( X_i \) the strategic variable that each firm chooses in period 1 as investment. The results to follow do not depend on which of the interpretations of cost (2.2a) or (2.2b) that one wishes to retain.

Combining equations (2.1) and (2.2) and redefining \( \alpha \) to be net of the linear cost term \( \gamma \), we obtain the expression below for firm profits,

\[
(2.3) \quad \Pi_i(q_i, X_i) = (\alpha - q_1 - q_2)q_i - \frac{(X_i - q_i)^2}{2} ; \quad i = 1, 2
\]

Before turning to the case where \( \alpha \) is random it is instructive to compare the differences in equilibrium investment and output for the deterministic case when it is and is not possible for firms to use prior investment to enhance their share of the market.

IIb. Comparison of Commitment and Non-Commitment Equilibria

In the noncommitment case, producers choose investment and output simultaneously. The firm's problem is to
subject to the investment-output decisions of the other firms. It is
easy to verify that there is a unique Nash equilibrium in this case where
\( x_1^0 = q_1^0 = \alpha / 3.7 \) (The superscript "0" refers to noncommitment equilibrium quantities).

In the commitment case each firm first chooses a level of investment,
\( X_i \), knowing that producers will make an output decision later on contingent
on the value of \( X = (X_1, X_2) \). Producing at levels that differ from \( X_i \) is
costly for firm \( i \), so that the choice of ideal capacity (or expenditure on
cost reducing technology) is a credible way for firm \( i \) to bind itself to
a specified range of production. We employ the perfect Nash equilibrium
concept in analyzing this case; we assume each firm calculates it's expected
production profits for given values of \( X \), and this knowledge is rationally
used by the firms in making their original investment decisions.\(^8\)

To compute the commitment equilibrium we first calculate each
firm's optimal output decision for given values of \( X \). Firm \( i \)'s output
choice is the solution to

\[
(2.5) \quad \maximize \: \Pi_i(q_i, X_i) \quad i = 1, 2
\]

where \( X_i \) is regarded as being fixed in (2.5). Differentiating (2.5)
with respect to \( q_i \) (assuming an interior solution) yields two equations
in two unknowns which can be used to solve uniquely for firm \( 1 \)'s and
firm 2's optimal production as a function of \( X_1 \) and \( X_2 \).

\[
(2.6a) \quad q_1^* = \frac{\alpha}{4} + \frac{3X_1}{8} - \frac{X_2}{8}
\]

\[
(2.6b) \quad q_2^* = \frac{\alpha}{4} + \frac{3X_2}{8} - \frac{X_1}{8}
\]

The "*" superscripts denote commitment equilibrium quantities).
Equation (2.6) reveals that the strategic use of investment increases
a firm's own output while reducing the optimal production of its com-
petitor.

Substituting for \( q_i \) from (2.6) into (2.5) we obtain an expression
for profits written in terms of the \( X \)'s.

\[
(2.7) \quad \Pi_i(X_i, X_j) = \frac{1}{128} \left[ 12\alpha^2 + 36\alpha X_i - 12\alpha X_j - 18X_i X_j - 37X_i^2 + 3X_j^2 \right] \quad i \neq j
\]

Assuming each firm chooses \( X_i \) to maximize profits taking as given the
investment, \( X_j \), of its competitor, we obtain (after some algebraic
manipulations) that the Nash equilibrium commitment levels of investment
are

\[
(2.8) \quad X_i^* = \frac{2m}{2 + m} = X_j^* \quad \text{where} \quad m = 18/37.
\]

Upon substituting for these values of \( X_1 \) and \( X_2 \) in (2.6) we find that
the equilibrium commitment output levels are

\[
(2.9) \quad q_1^* = \frac{\alpha (2 + 3m)}{4(2 + m)} = q_2^*
\]

Table 1 below provides a summary of the commitment and noncommitment
equilibrium quantities including firm profits, π, consumer surplus, CS, and total surplus, TS.9

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Comparison of Alternative Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non Commitment Eq.</td>
</tr>
<tr>
<td>q</td>
<td>.33α</td>
</tr>
<tr>
<td>X</td>
<td>.33α</td>
</tr>
<tr>
<td>π</td>
<td>.11α²</td>
</tr>
<tr>
<td>CS</td>
<td>.22α²</td>
</tr>
<tr>
<td>TS</td>
<td>.44α²</td>
</tr>
</tbody>
</table>

The table indicates that when commitment is possible, there is excessive investment, exposure, as each firm produces at a level less than its ideal capacity. Investment is necessary however for each firm to protect its share of the market. Predictably profits are less in the commitment equilibria because firms do not produce at peak efficiency, and the net revenue they earn from production only, is less than what they earn in the noncommitment equilibrium. Consumers benefit from the larger production associated with the commitment equilibria, and total surplus increases in this case as well.10

IIc Commitment Equilibria Under Uncertainty with Private Information

Now we assume that firms make investment decisions subject to some exogenous uncertainty about future demand or cost conditions. Producers' beliefs about future market conditions are captured in the form of a probability distribution for the intercept, α. Each firm i receives some additional information about α in the form of a signal which contains the results of a market survey, or a forecast of future cost conditions, etc. Firm i can observe its own signal costlessly, but it cannot acquire any additional information. We assume that each firm knows the mean and variance of its own signal, and the correlation (if any) between signal errors of different producers. Further knowledge of other players' prior beliefs about α or the distribution of their signals is not required for our model.

In period 1 each firm chooses a level of investment or capacity contingent on its signal. The signal received by each firm is used to update their information about α and to make inferences about the information that other producers have acquired. In period two, the actual value of α is revealed and the firms choose output to maximize profits given the X_i's. In analyzing the model we employ a Bayesian Equilibrium concept with the added requirement of perfectness. That is, each player uses his knowledge of the second period production equilibrium along with the probability assessments of future market conditions to rationally calculate his optimal choice of investment in period one. Bayesian Equilibrium requires that each producer chooses an investment function to maximize expected profits, contingent on a model of how it expects its opponents to behave. Each player's perceived model, and the true model of its opponents' behavior must coincide in equilibrium. The stochastic specifications of our model is as follows. There is some exogenous uncertainty about the demand intercept (or the linear cost term γ) which is reflected in a distribution for α which is specified as

\[(2.10) \quad α \sim N(\overline{α}, \sigma_α^2)\]
In most of what follows we assume that firms know the actual prior distribution for \( \alpha \), given by (2.10) though we will occasionally relax this by assuming that producers perceive the variance of \( \alpha \) to be \( \sigma_{\alpha}^2 \neq \sigma_{\alpha}^2 \).

(Unless otherwise, \( \sigma_{\alpha}^2 = \sigma_{\alpha}^2 \) for \( i = 1, 2 \)). Each producer \( i \), receives a signal, \( \eta_i \), which is related to \( \alpha \) by the equation

\[
\eta_i = \alpha + \varepsilon_i \quad i = 1, 2
\]

The noise terms \( \varepsilon_1 \) and \( \varepsilon_2 \) are independent of \( \alpha \), and \( (\varepsilon_1, \varepsilon_2) \) has a multivariate normal distribution with mean \((0,0)\) and variance covariance matrix

\[
\Sigma = \begin{bmatrix}
\sigma_1^2 & \sigma_{1,2} \\
\sigma_{1,2} & \sigma_2^2
\end{bmatrix}
\]

In (2.12) we allow for the possibility that signals differ in their degree of precision, and that although firms receive separate signals, they may be statistically related. When \( \sigma_{1,2} > 0 \), the signals may consist of information about demand drawn from similar consumer groups, for example. When \( \sigma_{1,2} < 0 \), the information may be drawn from two populations of consumers that are affected differently by a common event.

For future reference we note that, given \( \eta_i \), the conditional expectations of \( \alpha \), and \( \eta_j \), \( (j \neq i) \) are given by (see DeGroot (1970; pp. 167 and pp. 55)).

\[
(2.13) \quad E(\alpha|\eta_i) = \bar{\alpha} + \lambda_i (\eta_i - \bar{\alpha}); \quad \lambda_i = \frac{\sigma_{\eta_i}^2}{\sigma_{\eta_i}^2 + \sigma_{\alpha}^2} \quad i = 1, 2
\]

\[
(2.14) \quad E(\eta_j|\eta_i) = \bar{\alpha} + \gamma_j (\eta_i - \bar{\alpha}); \quad \gamma_j = \frac{\sigma_{\eta_j}^2 + \sigma_{\eta_j}^2}{\sigma_{\eta_j}^2 + \sigma_{\alpha}^2} \quad i \neq j
\]

Notice that the firms in our model obtain information which is unbiased; on average they hold correct perceptions about market conditions. Our model is reasonably general in that it allows for players to possess different degrees of informativeness, to process information in different ways when their priors on \( \alpha \) are not the same, and to receive independent or correlated signals about future market conditions. Our assumption that information flows are normally distributed greatly simplifies the analytics to follow, and provides a convenient way to characterize different information environments by the variance-covariance terms.

One problem with the normality assumption is that it permits instances when \( \alpha < 0 \), and/or \( \eta_1 \) or \( \eta_2 < 0 \). For simplicity we assume interior solutions to the firm's maximization problems. This means that we can end up with negative outputs or prices for certain realizations of \( \alpha \), \( \eta_1 \) and \( \eta_2 \). Although this is a conceptual problem, the probability of its occurrence can be made as small as one likes by limiting the size of the variance terms.
To derive the equilibrium investment functions we first note that second period production contingent on given realizations of \( X_1 \) and \( X_2 \) is given by equation (2.6). As before, substituting for \( q_1 \) and \( q_2 \) from (2.6) in (2.3) yields the expression for profits as a function of given values for \( X_1 \) and \( X_2 \) in (2.7). Our equilibrium consists of firms choosing functions \( X_i(\eta_i) \) to maximize expected profits conditioned on observing the signal \( \eta_i \). Formally, \( X_i(\eta_i) \) is derived as the solution to

\[
\text{maximize } \frac{1}{128} \left[ 12E(a^2|\eta_i) + 36E(a|\eta_i)X_i - 12E(aX_i|\eta_i) \right.
\]

\[
- 18E(X_j|\eta_i)X_i - 37X_i^2 + 3E(X_i^2|\eta_i) \right] \quad i \neq j
\]

\[
= \frac{1}{2} \left[ a_0 + a_1(\alpha_i + \eta_i) \right] \quad i = 1, 2
\]

Assuming an interior solution to (2.15), the investment functions for firms 1 and 2 satisfy

\[
X_1(\eta_1) = m \left[ E(a|\eta_1) - \frac{1}{2} E(X_2|\eta_1) \right], \quad m = \frac{18}{37}
\]

\[
X_2(\eta_2) = m \left[ E(a|\eta_2) - \frac{1}{2} E(X_2|\eta_2) \right]
\]

For now we posit that each producer perceives that its opponent uses a linear investment rule. This is in fact true in equilibrium as we will verify shortly. The perceived rules are given by

\[
X_1(\eta_1) = a_0 + a_1\eta_1
\]

\[
X_2(\eta_2) = b_0 + b_1\eta_2
\]

Bayesian equilibrium requires that each firm is aware of its opponent's rule for choosing \( X \). In this case (2.16) and (2.17) imply

\[
X_1(\eta_1) = m \left[ E(a|\eta_1) - \frac{1}{2} \left[ b_0 + b_1(a + \gamma_1(\eta_1 - \omega)) \right] \right]
\]

\[
X_2(\eta_2) = m \left[ E(a|\eta_2) - \frac{1}{2} \left[ a_0 + a_1(\alpha + \gamma_2(\eta_2 - \omega)) \right] \right]
\]

Upon substituting (2.13) and (2.14) into (2.18) we obtain

\[
X_1(\eta_1) = m \left[ \alpha + \lambda_1(\eta_1 - \omega) - \frac{1}{2} \left[ b_0 + b_1(\alpha + \gamma_1(\eta_1 - \omega)) \right] \right]
\]

\[
X_2(\eta_2) = m \left[ \alpha + \lambda_2(\eta_2 - \omega) - \frac{1}{2} \left[ a_0 + a_1(\alpha + \gamma_2(\eta_2 - \omega)) \right] \right]
\]

Now for a Nash Equilibrium to exist, the perceived decision rules in (2.17) must coincide with the actual decision rules given by (2.19). According to (2.19) this requires

\[
X_1(\eta_1) = m \left[ \alpha + \lambda_1(\eta_1 - \omega) - \frac{1}{2} \left[ b_0 + b_1(\alpha + \gamma_1(\eta_1 - \omega)) \right] \right]
\]

\[
X_2(\eta_2) = m \left[ \alpha + \lambda_2(\eta_2 - \omega) - \frac{1}{2} \left[ a_0 + a_1(\alpha + \gamma_2(\eta_2 - \omega)) \right] \right]
\]

for all values of \( \eta_1 \) and \( \eta_2 \). There exists a unique set of values \( (a_0, b_0, b_1) \) that satisfy...
Proposition 2.1. Given our assumptions, there exists a Bayesian Nash equilibrium characterized by (2.19) where

\[
(2.21) \quad a_1 = \frac{m(\lambda_1 - \frac{m}{2} \lambda_2 \gamma_1)}{1 - m^2 \gamma_1 \gamma_2} \quad \frac{m^2 \gamma_1 - m \lambda_2}{4}
\]

\[
a_0 = \max(1-\gamma_1) - \frac{m^2 \gamma_1 - m \lambda_2}{2} + m^2 \gamma_1 - \gamma_2 a_1 - \frac{m^2 \gamma_1 - m \lambda_2}{2} \gamma_1 b_1
\]

and \(b_1\) and \(b_0\) are defined symmetrically.

It is now apparent why we invoked the assumptions of linear demand quadratic costs and normality; together they allow us to derive linear closed form solutions for the equilibrium investment functions. It should be apparent from our development above that in fact linear decision rules occur in equilibrium whenever any linear updating rules are used for processing new information.

Furthermore, it turns out that the Bayesian equilibrium is unique. To demonstrate this we follow the same line of proof as in Novshek and Sonnenschein (1982, pp. 216). Let \(X_1(\eta_1) = a_0 + a_1 \eta_1, X_2(\eta_2) = b_0 + b_1 \eta_2\) and suppose \(\hat{X}_1(\eta_1)\) and \(\hat{X}_2(\eta_2)\) are other investment rules. Suppose \(\hat{X}_1(\eta_1)\) is best against \(\hat{X}_2(\eta_2)\). Then according to (2.16) we have

\[
(2.22) \quad \hat{X}_1(\eta_1) = m \left[ E(a|\eta_1) - \frac{1}{2} E \hat{X}_2(\eta_2|\eta_1) \right]
\]

Subtracting \(X_1(\eta_1)\) from both sides of (2.22) and using (2.16) yields

\[
(2.23) \quad \hat{X}_1(\eta_1) - X_1(\eta_1) = -\frac{m}{2} \left[ E \hat{X}_2(\eta_2|\eta_1) - E \hat{X}_2(\eta_2|\eta_1) \right]
\]

\[
= -\frac{m}{2} \left[ E(\hat{X}_2(\eta_2) - X_2(\eta_2|\eta_1) \right]
\]

Now it is apparent that

\[
(2.24) \quad \max_{\eta_1} \left| \hat{X}_1(\eta_1) - X_1(\eta_1) \right| \leq \frac{m}{2} \max_{\eta_2} \left| \hat{X}_2(\eta_2) - X_2(\eta_2) \right|
\]

But repeating the same argument, assuming \(\hat{X}_2(\eta_2)\) is best against \(\hat{X}_1(\eta_1)\), we obtain

\[
(2.25) \quad \max_{\eta_2} \left| X_2(\eta_2) - X_1(\eta_1) \right| \leq \frac{m}{2} \max_{\eta_1} \left| X_1(\eta_1) - X_1(\eta_1) \right|
\]

Together (2.24) and (2.25) imply \(\hat{X}_1(\eta_1) = X_1(\eta_1)\) so that we have established:

Proposition 2.2: Given our assumptions, the Bayesian Equilibrium characterized by (2.20) and (2.21) is unique.

The empirical predictions of our model are examined in detail in Sections III and IV. However, there are a few quick observations worth noting at this point. First, it is easy to verify that expected firm profits are not necessarily increasing with firm size as measured by investment or output. The reason is that \(X_1\) and \(q_1\) are typically increasing functions of the firm's signal, \(\eta_1\). A high value of \(\eta_1\) will be good news to firm 1 because it will imply a large demand on average. However, if demand is large, it is likely that firm 2 will also receive an optimistic signal, and that they will invest and produce on a large scale which will reduce firm 1's profits. Given these two opposing forces, it is not generally possible to determine the net impact of more optimistic signals on conditional expected profits.

Second, an interesting feature of our model is that variations in the stochastic environment produced by changes in \(\sigma_1, \sigma_2, \sigma_3, \sigma_4\) and \(\sigma_1, \sigma_2\) have no effect on equilibrium investment and output in an expected value sense. This property is due to our linear decision
III SYMMETRIC CASE

This section and the one to follow examine how market structure and performance are affected by the accuracy and the degree of overlap there is in the information firms receive. We assume that firms only process, but do not choose the amount or type of information that they observe. However, in reality the information environment that firms work in is not exogenous but it is partially shaped by public policy. Also, in some industries information flows are collectively controlled by the firms themselves, through their advocacy of disclosure laws, the material they provide to trade journals, and their participation and influence on external and self policing regulatory bodies. The results to follow include some predictive content about the information regimes we are likely to observe and some implications for policies for manipulating the structure and flow of information to improve market performance. The remainder of this section investigates situations where all market participants have equal access to information. Section IV examines the effects of information asymmetries on market organization and performance.

To begin, we assume that firms have the same prior beliefs about $a$ and they received identically distributed signals. For this situation firms 1 and 2 use the same investment function,

$$X_i (\eta_i) = a_0 + a_1 \eta_i \quad i = 1, 2$$

where

$$a_0 = a \left[ \frac{2m}{2+m} - \frac{2m\lambda}{2+m\gamma} \right] \quad a_1 = \frac{2m\lambda}{2+m\gamma}$$

Hence, on average, investment and output decisions are unaffected by the introduction of uncertainty into our model of commitment. The same equivalence result holds when we introduce uncertainty into our noncommitment model. Further, it can be shown that (details are not presented here) the qualitative differences between the commitment and noncommitment equilibria as indicated in Table 1 remain under conditions of uncertainty.
and \( \lambda = \sigma_{r_1}^2/(\sigma_{r_1}^2 + \sigma_{r_2}^2) \) and \( \gamma = (\sigma_{r_1}^2 + \sigma_{r_2}^2)/(\sigma_{r_1}^2 + \sigma_{r_2}^2) \) are common to both firms. Note that \( \sigma_{r_1}^2 \) is the variance of \( r \) as perceived by the firms. We will assume that although firms know the mean of \( r \), they may for example, have relatively diffuse priors on \( r \) if their initial information is poor. Normally however we will assume \( \sigma_{r_1}^2 = \sigma_{r_2}^2 \).

It is easy to verify that \( \sigma_{r_0}^2 = \sigma_{r_1}^2 \) are nonnegative. Furthermore, from Proposition 2.3 we know that \( \sigma_{r_0}^2 + \sigma_{r_1}^2 = \varepsilon(1) \equiv X \). A typical investment function is depicted in Fig. 1a. While the mean value of investment is unaffected by variations in \( \sigma_{r_1}^2, \sigma_{r_2}^2 \) and \( \sigma_{r_2}^2 \), the coefficients of the investment function \( \theta_0 \) and \( \theta_1 \) are nontrivial functions of these parameters, and hence do change with parameter shifts. Thus shifts in \( \sigma_{r_1}^2, \sigma_{r_2}^2 \) and \( \sigma_{r_2}^2 \) cause rotations of the investment function around the point \((X, \alpha)\) as depicted in Fig. 1b.

In markets where \( \text{var}(X_1 - X_2) \) is large we are more likely to observe firms of different sizes with one producer controlling most of the sales. Expost, one could infer that this market is dominated by a leading firm who had a first mover advantage. In contrast to this view, we are trying to identify stochastic environments that produce asymmetric market equilibria when producers start out being strategically equivalent. We note from above, that the observed differences in firm sizes as measured by investment or capacity are apt to be greater than the resulting inter-firm difference in market production shares.

The equations for our five variables of interest are given by (3.2) and (3.3)

\[
\text{var}(X_1 - X_2) = 2\alpha_1^2 (\sigma_{r_2}^2 - \sigma_{r_2}^2)
\]

(3.4) \( \bar{\pi} = \frac{1}{128} \left[ 12(\alpha_2^2 + \alpha_3^2) + 24(\alpha_2 + \alpha_3) + 52(\alpha_2^2 + \alpha_3^2) - 34 \alpha_1^2 \sigma_{r_2}^2 - 18 \alpha_1^2 \sigma_{r_2}^2 \right] \)

(3.5) \( \bar{\sigma_5} = \frac{1}{128} \left[ 16(\alpha_2^2 + \alpha_3^2) + 32(\alpha_2 + \alpha_3) + 16\alpha_2^2 (\sigma_{r_2}^2 + \sigma_{r_2}^2) + 8\alpha_2^2 (\sigma_{r_2}^2 + \sigma_{r_2}^2) + 16\alpha_2^2 \right] \)

(3.6) \( \bar{T} = \frac{1}{2} \bar{\pi} + \bar{\sigma_5} \)

The expressions for \( \bar{\pi}, \bar{\sigma_5} \) and are derived in Appendix A.

IIIa Affects of Increases in the Perceived and Actual Variance of \( r \)

Here we consider the comparative static effects of changes in the actual variance, \( \sigma_{r_1}^2 \), and the perceived variance, \( \sigma_{r_1}^2 \), of \( r \) on our variables of interest. In all but one instance, we assume that producers know the actual distribution of \( r \) and that \( \sigma_{r_1}^2 = \sigma_{r_2}^2 \). Our results are summarized in:
Proposition 3.1 Given our assumptions,

a) \( \frac{d\alpha}{\sigma_\alpha^2} > 0 \); \( \lim_{\sigma_\alpha^2 \to 0} \alpha_1 = 0 \); \( \lim_{\sigma_\alpha^2 \to \infty} \alpha_1 = 0 \)

b) \( \frac{d\text{var}(X_1 - X_2)}{\sigma_\alpha^2} > 0 \)

c) \( \frac{d\pi}{\sigma_\alpha^2} > 0 \)

d) \( \frac{d\pi}{\sigma_\alpha^2} \bigg|_{\sigma_\alpha^2 = \sigma^2} < 0 \)

e) \( \frac{d\text{CS}}{\sigma_\alpha^2} > 0 \)

f) \( \frac{d\text{TS}}{\sigma_\alpha^2} > 0 \)

The proof of Proposition 3.1 appears in Appendix B. The rest of our comparative static results which, like Proposition 3.1, are proved by straightforward differentiation of (3.2)-(3.6) will be stated without proof. Details of the proofs are available from the authors.

Part a of the Proposition is schematically represented in the Figure below. When \( \sigma^2_\alpha = 0 \), producers have precise knowledge of \( \alpha \). Hence the signal \( \eta_i \) is uninformative and it is ignored by the firms when choosing investment. As \( \sigma^2_\alpha \) increases, prior knowledge of \( \alpha \) becomes less precise, and the firms pay greater attention to their signals in that their investment schedules become more responsive to variations in their signals.

According to part b we are more likely to observe significant differences in firm size, and a higher concentration of production going to the largest firm, in markets that are subject to more demand uncertainty. The intuitive explanation is that as \( \sigma^2_\alpha \) increases, firms place greater importance on their private signals in choosing investment. To the extent that producers receive different information from their signals they are more likely to select different firm sizes.

Part c indicates that expected profits are larger in markets subject to more exogenous uncertainty. Together part b and part c also suggests that one is likely to find a correlation between market concentration as measured by differences in firm sales (or size) and average profitability.

Part d locally characterizes how expected profits vary as the firms' prior beliefs about \( \alpha \) become less precise. A lack of experience or errors in processing data may explain why firms don't know the actual distribution of demand. Expected profits (calculated using the true variance \( \sigma^2_\alpha \)) fall at least over
a small range, if producers lack confidence in their prior estimates of \( \alpha \) so that \( \sigma^2_\alpha > \sigma^2_\alpha \). On the other hand, expected profits will be higher when firms mistakenly ascribe too much precision to their estimates of \( \alpha \).

Parts e and f show that expected consumer surplus and total surplus increase with greater uncertainty (assuming \( \sigma^2_\alpha = \sigma^2_\alpha \)). Part e is reminiscent of the result in the economics of uncertainty literature, that consumer surplus increases with greater variation in demand.

IIIb Affects of Variations in Signalling Precision and Correlation

Our information structure is characterized by the precision of the demand signal and the degree of correlation there is between the information received by the independent firms.

In order to isolate the effects of varying signal precision on market equilibria we first assume there is zero correlation in the error terms, \( \sigma_{12} = 0 \). In this instance, firm specific information on demand is being drawn from independent populations of consumers. As \( \sigma^2_e \) ranges from zero to infinity the signal goes from identifying \( \alpha \) exactly to being completely uninformative. The effects of variations in the information content of the signal are summarized in

**Proposition 3.2**: Given our assumptions, and that \( \sigma_{12} = 0 \),

- \( \frac{\partial a_1}{\partial \sigma^2_e} < 0 \)  
  \( \lim_{\sigma^2_e \to \infty} a_1 = 0 \); \( \lim_{\sigma^2_e \to 0} a_0 = 0 \)

- \( \frac{\partial \text{var}(X_1 - X_2)}{\partial \sigma^2_e} \neq 0 \) as \( \sigma^2_e \neq \hat{\sigma}^2_e \);
  
  \( \lim_{\sigma^2_e \to 0} \text{var}(X_1 - X_2) = 0 \); \( \lim_{\sigma^2_e \to \infty} \text{var}(X_1 - X_2) = 0 \)

- \( \frac{\partial \text{var}(X_1 - X_2)}{\partial \sigma^2_e} \neq 0 \) as \( \sigma^2_e \neq \hat{\sigma}^2_e \);
  
  \( \lim_{\sigma^2_e \to 0} \text{var}(X_1 - X_2) = 0 \); \( \lim_{\sigma^2_e \to \infty} \text{var}(X_1 - X_2) = 0 \)

- \( \frac{\partial \text{TS}}{\partial \sigma^2_e} \neq 0 \)

As expected, \( a_1 \) decreases as the signal becomes less informative. According to part a of the proposition the signal is ignored by the firms in making investment decisions in the limit as \( \sigma^2_e \) tends to infinity.

The \( \text{var}(X_1 - X_2) \) is first increasing than decreasing as a function of \( \sigma^2_e \) according to part b. In particular there is a critical amount of signal noise, \( \hat{\sigma}^2_e \) which will generate the largest observed interfirm differences in investment and production. Note that extremely noisy signals do not produce large expect differences between producers, because they ignore the signals and make decisions based solely on prior information that they have in common about the distribution of \( \alpha \).

Part c implies that the firms are collectively better off receiving information with some amount of imprecision. This means that the collective value of better information may be negative; if both firms must receive signals with the same degree of precision, then, if possible, they will opt for at least some noise in information they acquire. In section IV we examine the value of information to a single firm who is able to unilaterally increase the precision of his signal.

As in the previous Proposition, parts b and c suggest a correlation between industry profitability and concentration as measured by inter-firm differences in investment or production.

Finally, we see that consumers are made worse off as producers receive less informative signals. Note that parts d and e indicate that while there
might be public support for improving the information flows in the market
the value of better information to the firms might well be negative.

The other factor affecting equilibria is the correlation between
information signals the firms receive. The signal on \( \alpha \) that each firm
observes is obscured by an error term, \( e_i \). When \( \sigma_{12} > 0 \) both firms may be
obtaining information by sampling from the same or similar populations. In the
negative correlation case producers may be sampling from groups that are
affected in opposite ways by a common occurrence (for example, a rise in
the price of gasoline may cause opposite demand responses in the domestic
and foreign travel industries). The correlation coefficient between \( \gamma_1 \) and
\( \gamma_2 \) is given by
\[
\rho = \frac{\sigma_{12}}{\sigma_\alpha \sigma_e} \quad \text{where} \quad |\sigma_{12}| < \sigma_e^2.
\]
As \( \sigma_{12} \)
becomes positive and large in absolute value, two things happen. First, both firms tend to observe the same signal. In a loose sense the total
information available for updating estimates of \( \sigma_\alpha^2 \) decreases. This is
made more precise when we analyze the effects of information pooling between
two firms. Second, since the firms are strategically equivalent, differences in information are the only source of variation in their investment-output
decisions. As their signals become more positively correlated the firms
expose decisions become more alike, and they tend to mimic each other. As
\( \sigma_{12} \) tends to \( -\sigma_e^2 \), the correlation between signals diminishes and may be
negative if \( \sigma_e^2 \) is large relative to \( \sigma_\alpha^2 \). In this case the decisions of the
producers may become uncorrelated or even negatively correlated. The effect
of these changes on our summary variables is characterized in

\[ \frac{d\varnothing(\gamma_1 - \gamma_2)}{d\sigma_{12}} < 0 \]

\[ \frac{d\varnothing}{d\sigma_{12}} < 0 \]

\[ \frac{d\Delta S}{d\sigma_{12}} < 0 \]

The results of part a and particularly part b are expected. Part a
implies that producers become more cautious in acting on new information as
their signals become better correlated. A producer observing a high value
for \( \eta \) is less apt to invest heavily because he suspects that it is likely
that his competitor has also probably observed an optimistic demand signal.

Parts b and c again suggest that industry profitability and con-
centration are correlated. In this instance there seems to be a direct link
between the variance of the differences in firm sizes and the magnitude of
our welfare measures. It can be shown that as \( \sigma_{12} \) increases, the variation in
total investment and total output increases. The reason for this is that
firms mimic each other's decisions as their signals become better correlated.
Hence they reinforce each other's errors, so that an unusually optimistic
(pessimistic) demand signal will result in both firms over (under) investing
in plant capacity. Recall that according to Proposition 2.2 the expected
value of total output is invariant to changes in \( \sigma_{12} \). This means that
an increase in signal correlation produces a mean preserving spread of the
distribution of total output. Hence expected total revenues and expected
consumer surplus will decrease because they are concave functions of total
output.

Proposition 3.3: Given our assumptions,

\[ \frac{d\sigma_1}{d\sigma_{12}} < 0 \]
Another way to view the role of signal correlation in our model is that all parties involved benefit when producers coordinate their decisions. If producer 1 plans for a large output, producer 2 should plan for a small one, and vice-versa so as to reduce unnecessary variations in total supply. This synchronization occurs automatically when the demand signals are negatively correlated. A more dramatic illustration of this principle is a four-way traffic light. Motorists heading in the north-south direction and those going in the east-west direction must synchronize; one group goes while the other group is stopped in order to avoid collision or costly delay. The traffic light provides the two groups of motorists with perfect negatively correlated signals.

One implication of our discussion above is that poor information is most troublesome to producers when their signals are positively correlated. Misleading signals about demand cause both firms to err in the same direction. When firms receive independent or negatively correlated information the impact of increases in signal variance is less clear as implied by Proposition 3.2 and our preceding discussion. The next two Propositions compare the impact of increasing signal noise on equilibria for two polar cases where the signal errors are perfect positively and negatively correlated.

**Proposition 3.4**: Given our assumptions and \( \sigma_{12} = \sigma_e^2 \) for all values of \( \sigma_e^2 \).

\[
\begin{align*}
\text{a.} & \quad \frac{d \pi_1}{d \sigma_e^2} < 0 \\
\text{b.} & \quad \frac{d \text{var}(X_1 - X_2)}{d \sigma_e^2} = 0 \\
\text{c.} & \quad \frac{d \pi_2}{d \sigma_e^2} < 0 \\
\text{d.} & \quad \frac{d \pi_{12}}{d \sigma_e^2} < 0 \\
\text{e.} & \quad \frac{d \pi_{21}}{d \sigma_e^2} < 0
\end{align*}
\]

**Proposition 3.5**: Given our assumptions, and \( \sigma_{12} = -\sigma_e^2 \) for all values of \( \sigma_e^2 \).

\[
\begin{align*}
\text{a.} & \quad \frac{d \pi_1}{d \sigma_e^2} < 0 \\
\text{b.} & \quad \frac{d \text{var}(X_1 - X_2)}{d \sigma_e^2} > 0 \\
\text{c.} & \quad \frac{d \pi_2}{d \sigma_e^2} > 0 \\
\text{d.} & \quad \frac{d \pi_{12}}{d \sigma_e^2} < 0 \\
\text{e.} & \quad \frac{d \pi_{21}}{d \sigma_e^2} < 0 \quad \text{for} \quad \sigma_{e}^2 \leq \frac{\sigma_e^2}{2} \quad ; \quad \sigma_{e}^2 > 0
\end{align*}
\]

Notice that the incentives for producers to jointly increase the informativeness of their signals depends on the degree of overlap in their information. Producer profits decrease with less signalling precision when they share common information. This suggests that producers may become better informed (assuming further information can be acquired) in markets where they have access to the same information. In such markets, information has public good characteristics because it benefits all parties involved. For that reason, information is likely to be underprovided if it is acquired privately by producers or by consumers who make it available (at no cost) to producers for their own use.

In contrast, the collective value of information to producers is always negative when signal noises are negatively correlated. Notice that, as before, \( \text{var}(X_1 - X_e) \) and expected profits vary together, and for the case at hand they are both decreasing as \( \sigma_e^2 \) gets small. This implies that producers are better off when they can maintain large systematic differences in their exposit investment-output decisions. If the firms are strategically equivalent
only differences in information can maintain a separation in decisions. Collectively, producers will resist programs to enhance the information they receive, whereas consumer groups will favour such attempts.

III.d Affects of Information Pooling

This section examines the incentives for producers to share private information with each other in order to obtain better demand estimates, assuming that they don't collude in making decisions. This analysis is motivated by the interesting work by Novshek and Sonnenschein (1982) on information pooling. They identify instances where sharing information is individually and jointly profitable for firms who can observe multiple independently distributed signals on demand. Our analysis differs most importantly from Novshek and Sonnenschein by allowing varying degrees of correlation to exist between the pooled signals. In particular we find that the incentives for sharing information diminish as the correlation between signals decreases and becomes negative.

Our results follow directly from Propositions 3.2 and 3.5 once we specify how information pooling transforms each firm's decision problems. First, a producer's estimate of demand becomes more precise when information is pooled (unless \( \sigma_{12} = \sigma_a^2 \) and the signals are identical, a possibility we shall ignore). For example, if \( \sigma_{12} = 0 \) so that the signal errors are uncorrelated, and firms pool their information then (see DeGroot p. 167) the updated distribution for \( \alpha \) is normal with mean equal to

\[
\lambda' = \lambda + \frac{\sigma_a^2}{\sigma_a^2 + \sigma_c^2/2} (\hat{\eta} - \bar{\gamma})
\]

and variance \( \frac{\sigma_a^2 \sigma_c^2/2}{(\sigma_a^2 + \sigma_c^2/2)} \). When firms can observe their own signal only, the conditional distribution of \( \alpha \) is normal with mean value given by (2.12) and variance equal to \( \frac{\sigma_a^2 \sigma_c^2}{(\sigma_a^2 + \sigma_c^2)} \). Comparing (2.12) and (3.7) we see that it is as if each firm is able to observe a signal with one half of the error variance when they share information.

If the signal errors are perfectly negatively correlated, the gains in estimation precision to information pooling are even more dramatic; the firms can exactly determine \( \alpha \) by computing

\[
\hat{\eta}^2 = \frac{\eta_1 + \eta_2}{2} = \frac{2\alpha}{2} = \alpha.
\]

A second effect of information pooling is to cause the ex post decisions of the firms to coincide. This combined with the first effect, that of increasing the precision of demand forecasts, determines the overall impact of sharing information. When \( \sigma_{12} = -\sigma_c^2 \) it follows from Proposition 3.4 that the impact of information pooling which eliminates demand uncertainty entirely, is characterized by

**Proposition 3.6:** Given our assumptions, and \( \sigma_{12} = -\sigma_c^2 \) then

(a) \( \eta^p > \eta \)

(b) \( \eta^p \sigma > \sigma \)

(c) \( \eta^p \sigma < \sigma \) for \( \sigma_c^2 < \sigma_c^2 \) ; \( \sigma_c^2 > 0 \)

(where the superscript "p" denotes quantities derived under a information pooling equilibrium).

When \( \sigma_{12} = 0 \), information sharing may increase profits by providing firms with more exact demand estimates, but it will simultaneously decrease profits by causing decisions to coincide. To determine the overall impact.
we compare equations (2.12) and (3.7). In effect when producers agree
to share information they move from a situation where \( \sigma_{12} = 0 \) and their
signal errors equals \( \sigma_e^2 \) to one where \( \sigma_{12} = \sigma_e^2 / 2 \), and the signal error
equals \( \sigma_e^2 / 2 \). If one totally differentiates the expressions for \( \bar{\pi}, \bar{CS} \)
and \( \bar{TS} \) with respect to \( \sigma_e^2 \) and \( \sigma_{12} \) allowing \( \sigma_e^2 \) to decrease from \( \sigma_e^2 \) to \( \sigma_e^2 / 2 \)
while \( \sigma_{12} \) increases at the same rate from \( \sigma \) to \( \sigma_e^2 / 2 \) and integrates, one
obtains the following characterization,

Proposition 3.7: Given our assumptions and \( \sigma_{12} = 0 \),

(a) \( \bar{\pi}^p < \bar{\pi} \)
(b) \( \bar{CS}^p > \bar{CS} \)
(c) \( \bar{TS}^p > \bar{TS} \)

Again we find that producers are worse off ex ante when they act on
common information obtained from pooling their signals. We have two
observations to make here. First, although there may be social gains to
allowing the firms to exchange information, producers prefer to maintain
private information if they are prohibited from colluding in making
investment and output decisions. It is at least conceivable that in a
second best world that producers should be allowed to make joint invest-
ment and output decisions to induce them to pool their information.

Second, we have only examined ex ante information pooling in which
producers agree to reveal their demand signals before knowing what they are.
Expost information sharing would allow the firms to first observe their
signal and then to decide whether or not they wanted to share it with
their competitors. This would amount to a simple type of information
transmission between firms. It would consist of a binary choice; a
firm would either send or not send a message to its competitor, and
if the message were sent it would simply contain the value of the demand
signal. This and other forms of strategic information exchange are
studied in our companion paper, Harris and Lewis (1982).
IV ASYMMETRIC CASE

Here we suppose that firms differ in how informed they are. One producer may receive a more precise demand signal than the other, or he may have a more accurate appraisal of the prior distribution for $\alpha$. In the first instance, it is common knowledge among the firms that one of them is better informed. In the latter case the less informed firm may not be aware of his relative ignorance. As before, we assume that producers hold correct expectations about the mean value of $\alpha$. Hence information asymmetries between the players are reflected by differences in the signal variances, $\sigma^2_{\varepsilon_i}$'s and each firm's subjective view of the dispersion of $\alpha$, denoted by $\overline{\sigma}_\alpha^2$. When $\sigma^2_{\alpha_i} = \sigma^2_{\alpha}$ then player $i$ is accurately informed about the true distribution of $\alpha$.

The natural question to ask here is what is the private and social value of information, as one firm becomes relatively better informed. Similar inquiries have been conducted by Novshek and Sonnenstein (1982) and by Palfrey (1982) as well as by other authors in the context of models of signalling, adverse selection, bidding and delegation. The interest in this subject was motivated by $a$, what is now no longer a novel result, that the private value of information may actually be negative. A crucial assumption for this result to hold is that information can not be freely disposed of, otherwise individuals could avoid harm from more information by simply ignoring it. In adversarial situations, this constraint on free disposal seems warranted. It may be difficult for one player to convince another that he will not use information he has acquired. Of course when information is only obtained at a cost, players can remain credibly uninformed by not purchasing information. The analysis to follow makes some comparisons of the ex ante profitability of differently informed firms assuming that information flows can not be altered. This is data that firms would need to know if they could affect their knowledge by acquiring information.

The expected profit for firms 1 and 2 for the case of asymmetric information is given respectively by

\[
\pi_1 = \frac{1}{128} \left[ 12(\alpha^2 + \sigma^2_{\alpha}) + 24(\overline{\alpha} \overline{X}) + (36a_1 - 12b_1)\sigma^2_{\alpha} - 54\overline{X}^2 - 18a_1 b_1 \sigma_{12} - 37a_1^2 \sigma^2_{\varepsilon_1} + 3b_1^2 \sigma^2_{\varepsilon_2} \right]
\]

\[
\pi_2 = \frac{1}{128} \left[ 12(\alpha^2 + \sigma^2_{\alpha}) + 24(\overline{\alpha} \overline{X}) + (36b_1 - 12a_1)\sigma^2_{\alpha} - 54\overline{X}^2 - 18a_1 b_1 \sigma_{12} - 37b_1^2 \sigma^2_{\varepsilon_2} + 3a_1^2 \sigma^2_{\varepsilon_1} \right]
\]

(These expressions are derived following the method outlined in Appendix A, assuming that the firms have different investment functions).

Our first result outlines the effects on the expected profits of the two firms of variations in the precision of one of the producer's signal. Totally differentiating (4.1) with respect to $\sigma^2_{\varepsilon_i}$ and $\sigma^2_{\varepsilon_j}$ and using (2.21) we obtain the following local results.

**Proposition 4.1** Given our assumptions,

a) If $\sigma^2_{\alpha_i} = \sigma^2_{\alpha}$ for $i = 1, 2$, then

(i) $\frac{d\pi_1}{d\sigma^2_{\varepsilon_i}} |_{\sigma^2_{\varepsilon_i} = \sigma^2_{\varepsilon_j}} < 0$

(ii) $\frac{d\pi_1}{d\sigma^2_{\varepsilon_j}} |_{\sigma^2_{\varepsilon_i} = \sigma^2_{\varepsilon_j}, i, j = 1, 2} > 0$
(b) If $\sigma_i^2 = \sigma_a^2$, $\sigma_j^2 = 0$, and $\sigma_{e_i}^2 > 0$ for $i, j = 1, 2$ then

(i) $\frac{d\pi_{e_i}}{d\sigma_{e_j}^2} > 0$

(ii) $\frac{d\pi_{e_j}}{d\sigma_{e_j}^2} < 0$

(c) If $\sigma_i^2 = \sigma_a^2$ and $\sigma_j^2 = +\infty$ then

(i) $\frac{d\pi_{e_i}}{d\sigma_{e_j}^2} > 0$

(ii) $\frac{d\pi_{e_j}}{d\sigma_{e_j}^2} < 0$

According to part (a) small asymmetries in information benefit the firm with the more precise demand signal. This also seems to hold when there are large differences in signal variances according to numerous numerical examples that we have looked at, though we haven't tried to establish this analytically because of the complicated algebra involved. Parts b and c deal with extreme cases; one in which one of the agents receives exact knowledge of demand by observing his signal, and another in which one of the players is initially completely uninformed about the distribution of demand. In both instances, the expected profit of the informed (uninformed) producer is increasing (decreasing in the signal variance of the uninformed firm. Proposition 4.1 suggests that in a model of information acquisition, becoming better informed than your opponent would be a dominant strategy. However as Propositions (3.2) and (3.5) suggest, producers might be made worse off if they both were to become better informed. The producers thus would be facing a type of

prisoner's dilemma in the acquisition of information.

Our final result illustrates that a small amount of ignorance about the prior distribution of demand can be beneficial for a producer. Specifically we have

**Proposition 4.2:** Given our assumptions, and $\sigma_i^2 = \sigma_a^2$ for $i = 1, 2$

$$\frac{d\pi_{e_i}}{d\sigma_i^2} > 0, \quad \frac{d\pi_{e_j}}{d\sigma_i^2} < 0 \quad i \neq j$$

According to Proposition 4.2 we suppose that the firms hold slightly different beliefs about the distribution of demand and that firm $i$ is misinformed in ascribing an overly diffuse prior to $a$. The divergence in beliefs may result because of interfirm differences in experience, and in processing information. In this case, ex ante expected profits are larger for the misinformed firm.\(^2\) This suggests that an informed firm could deliberately act as though it were uninformed about the prior distribution of $a$ in order to affect higher profits. This behavior is ruled out in the current analysis because we assume that firms are strategically equivalent and that one producer can not misrepresent his beliefs or preferences in order to achieve a more favorable outcome for himself.\(^2\)
V. CONCLUSIONS AND IMPLICATIONS

We have taken the approach that observed differences in firm size and market share may be explained by producers having access to different information at the time of their investment decisions. We have analyzed the equilibrium properties of a duopoly production model where each firm commits itself to a production plan based on proprietary information about future market conditions. Because our analysis pertains to a particular example, the conclusions we derive are regarded as only provisional, and are perhaps at least suggestive of what one might find in a more general analysis.

The primary implications of our model are:

1. Industry output and the capacity of each firm increases when firms can compete for market share by precommitting to a level of investment. Strategic precommitment may enhance overall industry performance, although it reduces productive efficiency.

2. One is likely to observe large ex post differences in firm size in industries where there is significant market uncertainty, and firms receive moderately accurate private estimates of demand which are uncorrelated or negatively correlated with each other. In these industries, we also expect to find a higher level of profits on average.

3. In the symmetric case, when firms are equally informed and they observe similar data, expected industry profits increase as information becomes more accurate. Receiving more exact information has the opposite effect on industry profits when the firms observe negatively correlated information signals. Overall, industry performance tends to improve with better access to information, so that an argument can be made for publicly provided information in this instance.

4. Under the conditions of our model, firms will resist sharing their private information with each other, when they can't collude on production. Instead, if possible, each firm will try to increase its stock of proprietary information, since better informed firms earn higher profits on average.

Our model can be extended in several directions to capture certain important real world complexities that we've ignored. First, it would seem straightforward to generalize the analysis to the case of \( N \geq 2 \) producers. Second, we could examine other modes of precommitment besides investment. For example, firms might select the degree of production flexibility based on their private forecasts of future conditions to secure a larger share of the market. Third, we might endogenize the acquisition of information as in Matthews (1979) and Milgrom (1981). Our current analysis suggests that there will be a proliferation of information gathering, possibly to the detriment of all producers, when investment in information becomes a strategic variable. Finally, the possibility that firms can exchange their private information ex post after observing it as in Crawford and Sobel (1982) is the subject of our companion paper.
APPENDIX A

Derivation of \( \text{Var}(X_1-X_2), \bar{\pi}, \overline{CS}, \) and \( \overline{TS} \) for the Symmetric Case

Derivation of \( \text{Var}(X_1-X_2) \)

\[
\text{VAR}(X_1-X_2) = \text{VAR}(a_1(n_1-n_2))
\]

\[
= a_1^2 \text{E}(n_1-n_2)^2
\]

\[
= a_1^2 E(n_1^2 + n_2^2 - 2n_1n_2)
\]

\[
= 2a_1^2(\sigma_\varepsilon^2 + \sigma_\alpha^2 - \sigma_\alpha^2 - \sigma_{12})
\]

\[
= 2a_1^2(\sigma_\varepsilon^2 - \sigma_{12})
\]

Derivation of \( \bar{\pi} \)

From (2.7) and the fact that \( X_1 = a_0 + a_1n_1 \), our expression for expected profit is

\[
\bar{\pi} = \frac{1}{128} \text{E} \left[ 12\alpha^2 + 36(a_0 + a_1(\alpha + \varepsilon_1)) - 12\alpha(a_0 + a_1(\alpha + \varepsilon_2)) \\
- 18(a_0 + a_1(\alpha + \varepsilon_1))(a_0 + a_1(\alpha + \varepsilon_2)) \\
- 37(a_0 + a_1(\alpha + \varepsilon_1))^2 + 3(a_0 + a_1(\alpha + \varepsilon_2))^2 \right]
\]

\[
= \frac{1}{128} \left[ 12(\alpha^2 + \sigma_\alpha^2) + 36\text{E}(a_0 + a_1\alpha) - 12\text{E}(a_0 + a_1\alpha) \\
- 18\text{E}(a_0 + a_1\alpha)^2 - 18a_1^2\text{E}(\varepsilon_1\varepsilon_2) \\
- 37\text{E}(a_0 + a_1\alpha)^2 - 37a_1^2\text{E}(\varepsilon_1^2) \right]
\]

Derivation of \( CS \)

Given \( q_1, q_2 \) and \( \alpha \) consumer surplus, \( CS \), is

\[
CS = \int_0^{q_1+q_2} (\alpha - Z) dZ - (\alpha - q_1 - q_2)(q_1 + q_2)
\]

\[
= \alpha(q_1 + q_2) - (q_1 + q_2)^2 - \alpha(q_1 + q_2) + (q_1 + q_2)^2
\]

\[
= \frac{(q_1 + q_2)^2}{2}
\]

\[
= \frac{1}{2} \left[ \frac{a}{4} + \frac{(X_1 + X_2)}{4} \right]^2 
\]  \text{(from 2.6)}

\[
= \frac{1}{2} \left[ \frac{a^2}{4} + \frac{\alpha(X_1 + X_2)}{4} + \frac{(X_1 + X_2)^2}{16} \right]
\]

\[
= \frac{1}{128} \left[ 16a^2 + 16\alpha(X_1 + X_2) + 4(X_1 + X_2)^2 \right]
\]

Now the expected value of \( CS \) is given by,
\( C_S = E(C_S) = \frac{E}{128} \left[ 16\alpha^2 + 16\alpha(2(a_0 + a_1\alpha) + a_1(c_1 + c_2)) + 4(2(a_0 + a_1\alpha) + a_1(c_1 + c_2))^2 \right] \)

\[ = \frac{1}{128} \left[ 16(\alpha^2 + \sigma_{\alpha}^2) + 32(\alpha X + a_1\sigma_{\alpha}^2) + 16(\alpha^2 + a_1^2\sigma_{\alpha}^2) + 8a_1^2(\sigma_c^2 + \sigma_{12}) \right] \]

**Derivation of \( TS \)**

It follows immediately that

\( TS = 2\pi + C_S \)

**APPENDIX B**

**Proof of Proposition 3.1**

**Part a**

\[ \frac{da_1}{d\sigma_{\alpha}^2} = 2m \left[ \frac{(2+\nu\gamma) \frac{d\lambda}{d\sigma_{\alpha}^2} - \lambda m \frac{d\gamma}{d\sigma_{\alpha}^2}}{(2+\nu\gamma)^2} \right] \]

\[ = \frac{2m \left[ (2+\nu\gamma) \sigma_{\gamma}^2 - \lambda m (\sigma_c^2 + \sigma_{12}) \right]}{(2+\nu\gamma)^2 - (\sigma_a^2 + \sigma_{c}^2)^2} \]

\[ = \frac{2m \left[ (2+m\sigma_{12}) \sigma_{\gamma}^2 + \lambda m \sigma_{12} \right]}{(2+\nu\gamma)^2 (\sigma_a^2 + \sigma_{c}^2)^2} \]

\[ = \frac{2m \left( 2\sigma_{\gamma}^2 + m(1-\lambda) \sigma_{12} + \lambda m \sigma_{12} \right)}{(2+\nu\gamma)^2 (\sigma_a^2 + \sigma_{c}^2)^2} \]

\[ = \frac{2m \left( 2\sigma_{\gamma}^2 + m \sigma_{12} \right)}{(2+\nu\gamma)^2 (\sigma_a^2 + \sigma_{c}^2)^2} = \frac{a_1^2}{2m(\sigma_{c}^2 + \sigma_{12})} (2\sigma_{\gamma}^2 + m \sigma_{12}) > 0 \]

**Part b**

\[ \frac{d\text{var}(X_1 - X_2)}{d\sigma_{\alpha}^2} = \frac{d}{d\sigma_{\alpha}^2} \left( 2a_1^2(\sigma_c^2 - \sigma_{12}) \right) \]

\[ = 2a_1(\sigma_c^2 - \sigma_{12}) \frac{da_1}{d\sigma_{\alpha}^2} > 0 \]
\[
\frac{d\sigma}{d\sigma_0} = \frac{1}{128} \left( (12+24a_1 - 52a_1^2) + \left[ (24 - 104a_1) \sigma_0^2 - 68a_1 \sigma_0^2 - 36a_1 \sigma_{12} \right] \frac{da_1}{d\sigma_0} \right)^2
\]

\[
= \frac{1}{128} \left( (12+24a_1 - 52a_1^2) \right)
\]

\[
+ \frac{a_1^2(2a_e^2 + m\sigma_{12})}{2m(\sigma_0^2 + \sigma_e^2)^2} \left[ (24 - 104a_1) \sigma_0^2 - 68a_1 \sigma_0^2 - 36a_1 \sigma_{12} \right]
\]

\[
= \frac{1}{128} \left( (12+24a_1 - 52a_1^2) \right)
\]

\[
+ \frac{2m(2\sigma_e^2 + m\sigma_{12})}{(2+m\psi)^2(\sigma_0^2 + \sigma_e^2)^2} \left[ (24 - 104a_1) \sigma_0^2 - 68a_1 \sigma_0^2 - 36a_1 \sigma_{12} \right]
\]

\[
= \frac{1}{128} \left( (12+24a_1 - 52a_1^2) \right) - 26.54 \left( \frac{(2m)(2\psi \mu' \sigma_{12})}{(2+m^2 \psi \mu' \sigma_{12})^2} \right)
\]

\[
\geq \frac{1}{128} \left( (12 + 24a_1 - 52a_1^2) - 26.54 \frac{(2m)}{(2+m^2 \psi \mu' \sigma_{12})} \right)
\]

\[
= \frac{1}{128} \left( (12 + 24a_1 - 52a_1^2) - 26.54 \frac{(2m)}{(2+m^2 \psi \mu' \sigma_{12})} \right)
\]

\[
= \frac{1}{128} \left( (12 + 24a_1 - 52a_1^2) \right) - 26.54 \frac{(2m)}{(2+m^2 \psi \mu' \sigma_{12})} \]

\[
= \frac{1}{128} \left( 12 - \max_{a_1} \left[ 2.54a_1 - 52a_1^2 \right] \right)
\]

\[
= \frac{1}{128} \left( 12 - 8.94 \right) > 0
\]

\[
\text{where } \psi = \sigma_0^2 / \sigma_e^2, \quad \psi' = \sigma_{12} / \sigma_e^2
\]

\[
\frac{d\sigma}{d\sigma_0} = \frac{1}{128} \left( (12+24a_1 - 52a_1^2) + \left[ 24 - \frac{(208m + 136\psi \mu' + 72m \psi')}{(2+m^2 \psi + m \psi')^2} \right] \right)
\]

\[
\geq \frac{1}{128} \left( (12+24a_1 - 52a_1^2) + \left[ 24 - \max \frac{(208m + 136\psi \mu' + 72m \psi')}{(2+m^2 \psi + m \psi')^2} \right] \right)
\]
Part d

\[
\frac{d\pi}{d\alpha} = \frac{1}{128} \left\{ (24-104\pi_1)\sigma_2^2+68\pi_1\sigma_2^2-36\pi_1^2 \right\} \frac{d\pi_1}{d\sigma_2^2} + \sigma_\pi^2 \frac{d\pi_1}{d\sigma_\pi^2}
\]

\[
\frac{1}{128} \left\{ \sigma_\alpha^2(24-208\frac{\sigma_\alpha^2}{\sigma_\epsilon^2} + 2 + 136\frac{\sigma_\epsilon^2}{\sigma_\alpha^2} + 72m\sigma_\alpha^2 \right\} \frac{d\pi_1}{d\sigma_2^2}
\]

\[
\frac{1}{128} \left\{ \sigma_\alpha^2 \frac{(48(\sigma_\alpha^2 + \sigma_\epsilon^2) + 24m(\sigma_\alpha^2 + \sigma_\epsilon^2) - 208m\alpha^2 - 136\frac{\pi_2^2}{\sigma_\alpha^2} - 72m\sigma_\alpha^2 \right\} \frac{d\pi_1}{d\sigma_\alpha^2}
\]

\[
\frac{1}{128} \left\{ \sigma_\alpha^2 \sigma_\epsilon^2 \frac{(41.42) - \sigma_\epsilon^2(65.1) - \sigma_\alpha(23.3) \right\} \frac{d\pi_1}{d\sigma_\alpha^2} < 0
\]

Part e

\[
\frac{d\pi}{d\alpha} = \frac{1}{128} \left\{ (16+32\pi_1 + 16\pi_1^2) + (32+32\pi_1)\sigma_2^2 + 16\pi_1(\sigma_2^2 + \sigma_\pi^2) \right\} \frac{d\pi_1}{d\sigma_2^2} + \sigma_\pi^2 \frac{d\pi_1}{d\sigma_\pi^2}
\]

> 0

Part f

\[
\frac{d\pi}{d\sigma_\alpha^2} = \frac{d\pi}{d\sigma_\alpha^2} + \frac{2d\pi}{d\sigma_\alpha} > 0
\]

Footnotes

1. For example see Salop (1981).
2. There are of course games where moving first is a liability. See Moulin (1981) for a characterization of order of play advantages in two person games.
3. The approach to explaining market evolution by looking to differences in private information among strategically equivalent firms instead of the first mover paradigm is also followed by Jovanovic (1981) in the context of a strategic location model and by Novshek and Sonnenschein (1982) in their study Cournot Duopoly with information pooling. A different approach to embellishing the existing first mover models suggested by Spulber (1981) and addressed by Applebaum and Lim (1982) is to assume that the firm moving first is partially uninformed about future market conditions.
4. Our analysis differs from Novshek and Sonnenschein (1982) in that we assume firms make a strategic commitment before they make output decisions.
5. Crawford and Sobel (1982) contain an interesting discussion of games of disclosure in a context quite different from ours.
6. Equation (2.2b) conforms to the assumptions used in a variety of models of R & D. See Dasgupta (1982) for a discussion of these models.
7. When \( x_1 \) and \( q_i \) are chosen simultaneously, our model reduces to the Novshek and Sonnenschein (1982) model. Looking at (2.3) it is apparent that selecting \( x_1 = q_i \) is optimal. Refer to Novshek and Sonnenschein (1982 pg. 216) for a proof that \( q_i = a/3 \).
8. In terms of the differential games literature, the noncommitment equilibrium corresponds to the players choosing "open loop" strategies whereas
"feedback strategies" are chosen in the perfect commitment equilibria.

The concept of perfectness is due to Setten (1975), an elaboration of the perfectness requirement in sequential games appears in Kreps and Wilson (1982).

9. The consumer surplus measure is taken as the area under the demand curve minus the total cost of purchasing \( q_1 + q_2 \) to the consumer.

10. The results that (a) investment and output increase, (b) profits decline and (c) consumer surplus rises as a result of commitment hold under rather general conditions. See Neary (1982).

11. The notion of Bayesian Equilibrium is due to Harsanyi (1968). Other names for this equilibrium concept are Self Fulfilling or Rational Expectations Equilibria.

12. When \( \sigma_{a_i}^2 \neq \sigma_{a_j}^2 \), \( \sigma_{a_i}^2 \) replaces \( \sigma_{a}^2 \) in the expressions for \( \lambda_1 \) and \( \lambda_2 \) in (2.18) and (2.14).

13. We are assuming that firms are risk neutral. Palfrey (1982) contains an interesting analysis of the impacts of risk aversion on equilibria in competitive situations.

14. The var \( (X_1 + X_2) = 2a_1(\sigma_a^2 + \sigma_{12}) \). Differentiating this expression with respect to \( \sigma_{12} \) reveals that var \( (X_1 + X_2) \) is increasing in \( \sigma_{12} \). The var \( (q_1 + q_2) = \frac{1}{16} \) var \( (X_1 + X_2) \).

15. This assumes that \( \sigma_{a_i}^2 = \sigma_a^2 \) for \( i = 1,2 \) and that firms do not also acquire some private information as in Novshek and Sonnenschein.

16. Exante profits are calculated as in (4.1) using the actual variance of \( \sigma_a^2 \).

17. See Sobel (1981) for an interesting discussion of this behavior in the context of bargaining.

18. See Novshek and Sonnenschein (1982) for cases where information pooling is mutually beneficial.

Bibliography


