ESTIMATION OF ECONOMETRIC MODELS
TWO-STAGE CONDITIONAL MAXIMUM LIKELIHOOD
I. INTRODUCTION

Estimation of Economic Models
TWO-STAGE CONDITIONAL MAXIMUM LIKELIHOOD

Abstract

We study two-stage conditional maximum likelihood estimation of economic models. This approach is valid even when the estimator is not available in closed form. We present the theoretical foundations of this method, including the derivation of the likelihood function and the estimation of model parameters. The results are illustrated through several applications in macroeconomics and finance.

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The methods proposed here are based on the fact that a causal model can be expressed as a system of ordinary differential equations. The differential equation for the i-th variable in the model is given by:

\[ \frac{d}{dt} x_i = \sum_j a_{ij} x_j + b_i \]

where \( a_{ij} \) are the coefficients relating the variables. The parameters \( a_{ij} \) and \( b_i \) are estimated from the data.

The problem of parameter estimation can be approached using various methods, such as least squares estimation or maximum likelihood estimation. The goal is to find the parameter values that best fit the observed data.

The paper also discusses the use of graphical models to represent causal relationships. Graphical models provide a visual representation of the causal relationships between variables, which can aid in the interpretation of the results.

In addition to parameter estimation, the paper also considers the issue of model selection. Model selection is the process of choosing the most appropriate model from a set of candidate models. This is done by comparing the goodness of fit of the models using various criteria, such as the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC).
Alternative, one can first maximize \( \mathcal{L} \) with respect to \( q \), then

\[
\frac{\partial \mathcal{L}}{\partial q} = 0
\]

Maximizing \( \mathcal{L} \) with respect to \( q \) leads to the following result:

\[
\mathcal{L} = \mathcal{L}(q, \theta) \quad \text{for all } q \text{ that maximize } \mathcal{L}
\]

where

\[
\mathcal{L}(q, \theta) = \mathcal{L}(q) + \mathcal{L}(\theta)
\]

Therefore, the optimal \( q \) that maximizes \( \mathcal{L} \) is

\[
q^{*} = \arg\max_{q} \mathcal{L}(q, \theta)
\]

In what follows, let \( q \) be the vector of parameters of \( \mathcal{L} \) not in \( \theta \).

For any two distributions over \( \mathcal{Y} \), the conditional distribution 

\[
\mathbb{P}(y|x) = \mathbb{P}(y|x, z) \mathbb{P}(z)
\]

Note that

\[
\mathbb{P}(y|x, z) = \mathbb{P}(y|x, z, \theta) \mathbb{P}(\theta|x, z)
\]

Therefore, the conditional distribution 

\[
\mathbb{P}(y|x) = \mathbb{P}(y|x, z) \mathbb{P}(z)
\]

is a density with respect to \( \mathbb{P}(x) \). Moreover, \( \mathbb{P}(x) \) is a lower semi-continuous function.

ASSUMPTION 3. \( \theta \) is a compact subset of \( \mathbb{R}^d \), and the section correspondence

\[
\tau(x) = \arg\max_{\theta} \mathcal{L}(x, \theta)
\]

and \( (\cdot, \cdot)^* \) are continuously differentiable on \( \mathbb{R}^d \) and \( \mathbb{R}^{d+1} \), respectively. If \( \mathcal{L} \) is a partition of \( \mathbb{R}^d \) into \( d+1 \) regions, then

\[
\mathcal{L}(x, \theta) = \mathbb{P}(y|x, \theta)
\]

for a fixed \( y \). Furthermore, for any \( \theta \) and \( \theta' \) in \( \mathbb{R}^d \),

\[
\mathbb{P}(y|x, \theta) = \mathbb{P}(y|x, \theta') \quad \text{if } \mathcal{L}(x, \theta) = \mathcal{L}(x, \theta')
\]

for all \( y \).
(1) \[
\begin{align*}
\int_{\mathbb{R}^d} P_{\mathbf{z}}(\mathbf{z}) \, d\mathbf{z} &= 1, \\
\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}, \\
\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}, \\
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\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}.
\end{align*}
\]

(2) \[
\begin{align*}
\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}, \\
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\end{align*}
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(3) \[
\begin{align*}
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\end{align*}
\]

(4) \[
\begin{align*}
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\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}, \\
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\int_{\mathbb{R}^d} P_{\mathbf{z} | \mathbf{y}}(\mathbf{z} | \mathbf{y}) \, d\mathbf{z} &= P_{\mathbf{y}}.
\end{align*}
\]
not be correctly specified. These properties are summarized in the following

General condition: $E(\theta' | y)$, the conditional moment for $y$. Given $y$, $
abla E(\theta' | y)$ need

We shall first derive the asymptotic properties of $\hat{\theta}_n$ under

3. ASYMPTOTIC PROPERTIES OF $\hat{\theta}_n$ ESTIMATOR

# assumption that $E(\theta' | y)$ and $E(\hat{\theta}' | y)$ are both non-regular.

# together with assumption (a): $\lambda_1(\theta)$ is positive and assumption (b): $\lambda_2(\theta)$ is positive and assumption (c): $\lambda_3(\theta)$ is finite and assumption (d): $\lambda_4(\theta)$ is finite and assumption (e): $\lambda_5(\theta)$ is finite and assumption (f): $\lambda_6(\theta)$ is finite and assumption (g): $\lambda_7(\theta)$ is finite and assumption (h): $\lambda_8(\theta)$ is finite and assumption (i): $\lambda_9(\theta)$ is finite and assumption (j): $\lambda_{10}(\theta)$ is finite.

Assumption (e): $\lambda_1(\theta)$ is an interior point of $\theta$.

# a regular point of $\theta$.

Let

$$
\frac{\partial \theta}{\partial \theta'} = \frac{\partial \theta}{\partial \theta'}
$$

and

$$
\lambda(\theta) = \lambda(\theta)
$$

where $\lambda(\theta)$ is the $k \times k$ matrix obtained from $\lambda(\theta)$. Let

$$
\frac{\partial \theta}{\partial \theta'} = \frac{\partial \theta}{\partial \theta'}
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and

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$$

where $\lambda(\theta)$ is the $k \times k$ matrix obtained from $\lambda(\theta)$. Let

$$\text{constant integral function associated with } f(Y' | \theta) - f(Y' | \theta)$$

members (1969), theorem 3, p. 65. Applet lo:
Given a sequence of IID random variables $X_1, X_2, \ldots, X_n$ with parameter $\theta$, suppose that $(X_1, \ldots, X_n)$ follows a distribution from a parameter space $\Theta$. If $\hat{\theta}$ is an estimator of $\theta$, then

$$\frac{\hat{\theta} - \theta}{\|\theta - \theta\|_2} \overset{\text{a.s.}}{\longrightarrow} 0.$$

**Theorem 1** (Approximate Properties of $\hat{\theta}$ under Weak Dependence)

The exact results follow from the central limit theorem (CLT). Thu, p. 5.5. Suppose that $\hat{\theta}$ is an estimator of $\theta$, then for some $\varepsilon > 0$, $\frac{\hat{\theta} - \theta}{\|\theta - \theta\|_2} \overset{\text{a.s.}}{\longrightarrow} 0.$

**Proposition 2** (Consistency of $\hat{\theta}$)

In the context of Section 6, suppose that $\hat{\theta}$ is an estimator of $\theta$ based on a set of independent observations. If $\hat{\theta}$ is consistent, then

$$\sqrt{n} (\hat{\theta} - \theta) \overset{\text{D}}{\longrightarrow} N(0, I)$$

for some $\varepsilon > 0$. This property is derived in the context of the asymptotic distribution of $\hat{\theta}$.

**Theorem 3** (Generalized Confidence Intervals)

For a general confidence level $1 - \alpha$, we can construct approximate confidence intervals based on the asymptotic properties of $\hat{\theta}$.

$$\left( \hat{\theta} - z_{1-\alpha/2} \frac{1}{\sqrt{n}}, \hat{\theta} + z_{1-\alpha/2} \frac{1}{\sqrt{n}} \right)$$

where $z_{1-\alpha/2}$ is the $1-\alpha/2$ quantile of the standard normal distribution.

**Corollary 4** (Limiting Distribution)

As $n \rightarrow \infty$, we have

$$\sqrt{n} (\hat{\theta} - \theta) \overset{\text{D}}{\longrightarrow} N(0, I).$$

**Proof**

The proof follows from the asymptotic properties of $\hat{\theta}$ and the central limit theorem (CLT).
In this section, we shall establish that the conditional model for

\[ (\mathbf{Y}^T_{12})_{T=1} \cdot \mathbf{Y}^T_{03} \cdot \mathbf{Y}^T_{04} \cdot \mathbf{Y}^T_{05} \cdot \mathbf{Y}^T_{06} \cdot \mathbf{Y}^T_{07} \]

only involves the parameter \( \mathbf{Y}^T_{03} \) over \( \mathbf{Y}^T_{04} \) and \( \mathbf{Y}^T_{05} \) over \( \mathbf{Y}^T_{06} \) and \( \mathbf{Y}^T_{07} \).

The conditional determinant of \( \mathbf{Y}^T_{12} \) can then be written as

\[ (\mathbf{Y}^T_{12}) = (\mathbf{Y}^T_{03}) (\mathbf{Y}^T_{04}) (\mathbf{Y}^T_{05}) (\mathbf{Y}^T_{06}) (\mathbf{Y}^T_{07}) \]

where the function of \( \mathbf{Y}^T_{03} \) and \( \mathbf{Y}^T_{04} \) and \( \mathbf{Y}^T_{05} \) and \( \mathbf{Y}^T_{06} \) are defined as in equation (1.4)

\[ (\mathbf{Y}^T_{03})^2 + (\mathbf{Y}^T_{04})^2 = (\mathbf{Y}^T_{05})^2 \]

In discussing the conditional model, we have

\[ (\mathbf{Y}^T_{05})^2 \]

defined as above. For instance, \( (\mathbf{Y}^T_{05})^2 \) corresponds to the function \( (\mathbf{Y}^T_{05})^2 \) as defined in equation (1.4).

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\[ (\mathbf{Y}^T_{03}) (\mathbf{Y}^T_{04}) (\mathbf{Y}^T_{05}) (\mathbf{Y}^T_{06}) (\mathbf{Y}^T_{07}) \]

only involves the parameter \( \mathbf{Y}^T_{12} \) over \( \mathbf{Y}^T_{03} \) and \( \mathbf{Y}^T_{04} \) over \( \mathbf{Y}^T_{05} \) and \( \mathbf{Y}^T_{06} \) over \( \mathbf{Y}^T_{07} \).

The conditional determinant of \( \mathbf{Y}^T_{03} \) can then be written as

\[ (\mathbf{Y}^T_{03}) = (\mathbf{Y}^T_{12}) (\mathbf{Y}^T_{04}) (\mathbf{Y}^T_{05}) (\mathbf{Y}^T_{06}) (\mathbf{Y}^T_{07}) \]

where the function of \( \mathbf{Y}^T_{12} \) and \( \mathbf{Y}^T_{04} \) and \( \mathbf{Y}^T_{05} \) and \( \mathbf{Y}^T_{06} \) and \( \mathbf{Y}^T_{07} \) are defined as in equation (1.4)

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\[
(\beta') \quad \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_k
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{n} X_i \\
\sum_{i=1}^{n} X_i y_i \\
\vdots \\
\sum_{i=1}^{n} X_i y_i x_i^k
\end{bmatrix}
\]

Theorem 3 (Approximate Efficiency of OLS): Given assumption A1-A6, the variance information matrix

\[
\begin{bmatrix}
\sigma^2 \\
\sigma^2 x_1 \\
\vdots \\
\sigma^2 x_1 x_k
\end{bmatrix}
\]

The variance-covariance matrix of is given by

\[
\begin{bmatrix}
\sigma^2 \\
\sigma^2 x_1 \\
\vdots \\
\sigma^2 x_1 x_k
\end{bmatrix}
\]

Lemma 3: Given assumptions A1-A6, A7-A9, all the following matrices exist.
From Equation (3.5), we have:

\[
\left[ \begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array} \right] - (\theta)A = (\theta)A - (\theta)^{2}Z_{A}
\]

The assumptions of Lemma 1 and Hypothesis 2, respectively, let

\[
\begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array}
\end{array}
\]

be the difference in the models for some positive integers \( k \) and \( m \). In other words, from the previous sections, the assumption (3.5) can be equivalently rewritten as

\[
\begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array}
\end{array}
\]

Theorem 2. Let the assumptions

\[
\begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array}
\end{array}
\]

Then, for some positive integers \( k \) and \( m \),

\[
\begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array}
\end{array}
\]

5. SOME TESTS FOR MODEL DIFFERENTIATION

In this section, we shall be interested in deriving some results of the form

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

The following question arises:

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

The first set of model differentiation results we consider is based on the assumption of correct restricted interaction.

Assumption 1.

In order to derive the restricted interaction of the present model that is obtained in the interaction matrix equation, one can attempt to take the interaction matrix equation, where the following two statements appear to be mutually exclusive:

\[
\begin{array}{c}
\begin{array}{c}
(\theta)^{2}Z_{A} \\
(\theta)^{2}Z_{A}
\end{array}
\end{array}
\end{array}
\]

Alternatively, results for model differentiation have been proposed (see

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

Therefore, it follows that the equation holds.

For the statistic to be applicable, in general, we have

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

For the statistic to be applicable, in general, we have

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

Theorem 2. Let the assumptions

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

Then, for some positive integers \( k \) and \( m \),

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]

Theorem 2. Let the assumptions

\[
(\theta)A + (\theta)^{2}Z_{A} = (\theta)^{2}Z_{A}
\]
there are more than one variable related to each other, and also

\[ \mathbf{H}_1 \cap \mathbf{H}_2 = \mathbf{H}_1 \]

For any choice of \( p \), the test statistic is given by

\[ Q = \mathbf{T}^\top \mathbf{H}_2 \mathbf{T} \]

when \( \mathbf{H}_2 \) is the saturated hypothesis.

From this theorem, it follows that the test statistic is independent of

\[ \mathbf{H}_1 \cap \mathbf{H}_2 = \mathbf{H}_1 \]

where \( \mathbf{H}_1 \) is the saturated hypothesis.

For any choice of \( p \), the test statistic is given by

\[ Q = \mathbf{T}^\top \mathbf{H}_2 \mathbf{T} \]

when \( \mathbf{H}_2 \) is the saturated hypothesis.

From this theorem, it follows that the test statistic is independent of

\[ \mathbf{H}_1 \cap \mathbf{H}_2 = \mathbf{H}_1 \]

where \( \mathbf{H}_1 \) is the saturated hypothesis.
THEOREM 5 (Gradual Termination): Given assumptions A1—A6, A7—A8, if

then there exists an l such that

The proof of this theorem is based on the assumption that the algorithm terminates in a finite number of steps. The proof involves the construction of a sequence of states, each of which is generated by applying a transition function to the previous state. The sequence terminates when a fixed point is reached, which corresponds to a state where no further changes can be made. The proof relies on the fact that the transition function is monotonic, meaning that it always moves from one state to a state that is greater in some sense. This ensures that the sequence must eventually reach a fixed point, which is the desired termination result.
XI. Estimation of the Parameters in the Coincident Model

[Text continues with mathematical expressions and proofs related to estimation of parameters in a coincident model.]
null hypothesis, a consistent estimator of the variance of a linear combination of the sample
variables is the sample variance of the linear combination. The test statistic is given by:

\[ t = \frac{\bar{X} - \mu}{s / \sqrt{n}} \]

where \( \bar{X} \) is the sample mean, \( \mu \) is the population mean, \( s \) is the sample
standard deviation, and \( n \) is the sample size.

In the multivariate case, the test statistic is given by:

\[ t = \frac{\bar{X} - \mu}{S / \sqrt{n}} \]

where \( \bar{X} \) is the sample mean vector, \( \mu \) is the population mean vector, \( S \) is the
sample covariance matrix, and \( n \) is the sample size.

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sample covariance matrix, and \( n \) is the sample size.

In both cases, the test statistic follows a t-distribution with \( n-1 \) degrees of freedom.
example 3 is to be tested.

the product form equation associated with the right-hand side variable.

1.2.5.2.1.3

The proof of Theorem 3.2.5.2.1.3 is deferred to the Total Appendix and
done by contradiction to the hypotheses set forth by the Total Equation and
deployed method of analysis for exponential of variances in the process. In the
second, this to

as in the previous example, the total procedure can also be used to

second stage

the (1.2.5.2.2)(1.2.5.2.2) and (1.2.5.2.2)(1.2.5.2.2) are in Heckman's
when it does not contain the cooperation of the model for $\hat{\alpha}$, then $\hat{\alpha}$ is not easy to obtain

estimation of the model for $\hat{\alpha}$ or $\hat{\alpha}$, respectively. For the
determinants one does not require to exchange (1.2.5.2.2) for (1.2.5.2.2)

therefore, our procedure has the following advantage:

in this stage. Our procedure differs from it in the second

throughout the entire method of estimation, this can be used. The

asymptotic normal under correct asymptotic assumptions, as hypotheses that can be

Theorem 2 ensures that the estimator (1.2.5.2.2) is consistent and

\[ \hat{\theta} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\hat{\theta}_i}{\hat{\theta}_i} \right) \]

the positive solution to the exponential equation. The positive solution to the

observations on the exponential equation. The vector of observations on $\theta$ is $\hat{\theta}_i$. The number of observations on $\theta$ in the

where $I_n$ is the number of observations that are $\theta$, $\hat{\theta}_i$ is the

$\hat{\theta}_i = I_n x_i - I_{n\theta} \hat{\theta}_i \hat{\theta}_i$.

second step is part of the asymptotic analysis of the

As the second product in (1.2.5.2.2) is dependent of the

unknown estimated covariance of the coefficient in the product analysis of the

Example 3: The previous examples deal with the multinomial case. The

Example 3: The previous examples deal with the multinomial case. The

Proof: The result follows from the multivariate version of the central limit theorem.

\[
\begin{pmatrix}
\frac{\mathbf{X} - \mathbf{E}X}{\mathbf{V}^{1/2}} \\
\frac{\mathbf{Y} - \mathbf{E}Y}{\mathbf{V}^{1/2}}
\end{pmatrix}
\sim
\begin{pmatrix}
\mathbf{E}X \\
\mathbf{E}Y
\end{pmatrix} +
\begin{pmatrix}
\mathbf{V}^{1/2} \\
\mathbf{V}^{1/2}
\end{pmatrix}
\begin{pmatrix}
\mathbf{Z} \\
\mathbf{Z}
\end{pmatrix}
\]

Theorem: Given assumption A1-A5-(a).

Following lemma.

\[
\mathbf{X} \sim \text{Normal} \left( \mathbf{E}X, \mathbf{V} \right)
\]

Following lemma.

\[
\mathbf{Y} \sim \text{Normal} \left( \mathbf{E}Y, \mathbf{V} \right)
\]

\[
\mathbf{Z} \sim \text{Normal} \left( \mathbf{0}, I \right)
\]

Following lemma.

\[
\mathbf{X} \sim \text{Normal} \left( \mathbf{E}X, \mathbf{V} \right)
\]

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\]

Following lemma.

\[
\mathbf{Z} \sim \text{Normal} \left( \mathbf{0}, I \right)
\]

To prove the asymptotic normality of \( \mathbf{Z} \), we use the

\[2.0, (5.0) \begin{pmatrix}
\mathbf{X} \\
\mathbf{Y}
\end{pmatrix}
\]

To prove the existence of \( \mathbf{Z} \), we use the

Appendix

7. Conclusion

In this paper, we considered a general method called two-stage

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\[
\text{The above construction, as a consequence of (1) and \( \theta \), results in a measurable function \( \lambda_{\theta} \) on \( \mathbb{T} \times [0, 1] \) such that for all \( (x, t) \in \mathbb{T} \times [0, 1] \), we have:}
\]

\[
\int_{\mathbb{T}} \lambda_{\theta}(x, t) \, dx = \theta(t)
\]

for almost every \( t \in [0, 1] \).
Given assumption of (q) of Theorem 1:3.7, the matrix \( \mathbf{A} \) and \( \mathbf{B} \) are non-singular. Then part (d) follows from Lemma 2.

\[ (\theta - \bar{\theta}) \mathbf{A}^{-1} \mathbf{B} \]

From assumption of (q) of Theorem 1:3.7, the matrix \( \mathbf{A} \) and \( \mathbf{B} \) are non-singular.

\[ (1)^{\theta} + (\theta - \bar{\theta}) \mathbf{A}^{-1} \mathbf{B} \]

Prove part (d). From Theorem 2 (1964).
proof of lemma 2: given assumption 2 as, the k × k matrix

\[
(\phi'(\theta)^T)^T (\phi'_{\theta}\theta) \cdot (\phi'(\theta)^T)^T (\phi'_{\theta}\theta)
\]

can be set to 0. then, for some θ, θ' = θ.

\[
(\phi'(\theta)^T)^T (\phi'_{\theta}\theta) = (\phi'(\theta)^T)^T (\phi'_{\theta}\theta)
\]

for some θ. hence, (θ) is correctly specified.

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0 \cdot 0 = 0
\]

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\[
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\[
0 \cdot 0 = 0
\]
To prove (a), let \( \Lambda \) be any \( \mathcal{M} \)-representative of \( H \), it follows from Proposition 1 that \( \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \frac{u}{u} \u2026
To prove the next result, the following lemma is used.

Lemma: Given assumptions of Lemma 2 and the perturbation structure of $\Theta_0$, we get the following equations:

\[
(1)\, d_0 + \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu = \frac{\tilde{z}_0 \theta - u_0}{\theta_0} \tau \mu - \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu
\]

Proof:

To prove part (a) we use the following equalizations for $\tilde{z}$:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

where we have used the assumption of the main theorem. The proof of the main theorem is shown:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

We obtain under certain assumptions:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

and $\theta_0$. From this equality, we have:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

For $\theta_0$ and $\tilde{z}$, we have:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

where we have used the assumptions of the main theorem. But from equation (a), we have:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

Then we have:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

and $\theta_0$. From equation (a), we have:

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where we have used the assumptions of the main theorem. But from equation (a), we have:

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\[
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(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]

Then we have:

\[
(1)\, d_0 + \left( \frac{\sum_{i=1}^{r} \tilde{z}_i \theta_i - u_0}{\theta_0} \tau_i \mu \right) = 0
\]
The following lemma is used to prove theorems.

Theorem (a). Assume the lemma of the proof of part (b).

The proof now proceeds as follows. It follows from the lemma (19.1) that the rank of the matrix A is equal to the rank of the matrix B. Therefore, the rank of the matrix A is equal to the rank of the matrix B.

Since the rank of the matrix A is equal to the rank of the matrix B, it follows that the matrix A is invertible. Therefore, the matrix A is invertible.

From the previous equation, we have

\[ (I^0 - \mathbf{u} \mathbf{z})^T \mathbf{z}^T \mathbf{z} = (I^0 - \mathbf{u} \mathbf{z})^T \mathbf{z}^T \mathbf{z} \]

and

\[ (I^0 - \mathbf{u} \mathbf{z})^T \mathbf{z}^T \mathbf{z} = (I^0 - \mathbf{u} \mathbf{z})^T \mathbf{z}^T \mathbf{z} \]

Thus, from Theorem (19.1), it follows that the rank of the matrix A is equal to the rank of the matrix B.

Therefore, the matrix A is invertible. Hence, the theorem is proved.
(4) The first part of part (a) is straightforward. It follows that the rank of matrix $\hat{A}$, which is $\hat{A}'\hat{A}$, is full rank.

Matrix $\hat{A}'\hat{A}$ can be decomposed into the product of two matrices, $\hat{A}_p\hat{A}_p{}^T$ and $\hat{A}_t\hat{A}_t{}^T$, where $\hat{A}_p$ and $\hat{A}_t$ are orthogonal matrices.

The second equation follows from an application of Theorem 4.1. The proof is as follows:

\[
(I)\hat{A}'\hat{A} + (\hat{A}_p\hat{A}_p{}^T + \hat{A}_t\hat{A}_t{}^T)\hat{A}_p = \hat{A}_p\hat{A}_p{}^T + \hat{A}_t\hat{A}_t{}^T
\]

where $\hat{A}_p$ and $\hat{A}_t$ are orthogonal matrices.

Then, using the normal equation for $\hat{A}_p\hat{A}_p{}^T$, we get

\[
(I)\hat{A}'\hat{A} + (\hat{A}_p\hat{A}_p{}^T + \hat{A}_t\hat{A}_t{}^T)\hat{A}_p = \hat{A}_p\hat{A}_p{}^T + \hat{A}_t\hat{A}_t{}^T
\]

Theorem 4.1.

Moreover, the result follows from the multivariate version of the central limit theorem.
\[ (1)^\theta_0 + \left( u_\theta_0 - u_\theta_0 \right) \mathbf{z}_1 \mathbf{u}(\theta_0) \mathbf{z}_2 \theta_0 \theta_4 = \]

\[ (1)^\theta_0 + \left[ \frac{\mathbf{z}_1 \mathbf{u}(\theta_0) \mathbf{z}_2 \theta_0 \theta_4}{1 - (\lambda_0 \mathbf{z}_1 \mathbf{u}(\theta_0) \mathbf{z}_2 \theta_0 \theta_4)} \right] \]
adaptable covariance matrices of the alternative need no longer be
parameterized (5.6)-(5.7) under the assumption. However, the
covariance matrices need not enter the behavior of the
alternatively, any alternative. Our results can also be evaluated at an
unrestricted margin to accommodate the adaptable covariance matrices under the assumption. Thus, the adaptable

covariance matrices allow the parameterization of the adaptable

For what follows, one needs only to consider only the adaptable

the preceding result.

8. The above results hold for a parameterization that is not the

evaluation space in which (9.1) and (9.2) (see Assumption 9.1)

when (9.3) is needed. The in (9.4), it is needed in (9.5),

matrix for (9.6) and (9.7) can also be expressed in terms of the

matrices (9.8) and (9.9) in Lemma 3.9, because of the information

matrix that is given in Lemma 3.10.

9. Note that if one assumes a statistical model to be homogenous, then

the metrizations in the model are equivalent with


4. For a definition of lower semi-continuity, see (6.1) (Berger, 1965)). I am

sharper results.

structure of the statistical model, this allows us to derive

though statement to write's method, take advantage of the

I, a related method was also considered by White (1982b). Our method,

in two or more groups, remaining errors of course, when

to pose hypotheses for testing. On the other side, however, that have been worked out

PREFACE
REFERENCES