EVIDENCE OF BLOCK SWITCHING IN DEMAND SUBJECT TO DECLINING BLOCK RATES - A NEW APPROACH*

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ABSTRACT

This paper considers the problem of forecasting demand subject to a non-linear rate schedule. We develop an empirical model of electricity demand subject to a quantity determined rate schedule and suggest a new procedure to estimate population taste variation. Using micro-level data from the 1975 Washington Center for Metropolitan Studies (WCMS) survey, we provide evidence on the prevalence and extent of block switching.
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I. Introduction

This paper considers the problem of forecasting demand subject to a non-linear rate schedule for commodities such as electricity, water, and tele-communications. We attempt to calculate the probability of block switching and resulting demand for non-marginal changes in price. We will determine whether consistent estimation corrections for price endogeneity are likely to be of much value and whether the movement from intra-marginal to marginal blocks is a significant determinant of price elasticity.¹

The research line motivating these issues is well tred. The first systematic discussion of price specification in conditional demand models was given by Houthakker (1951a). This was followed by Taylor (1975) and Nordin (1976) who introduced the distinctions between marginal and average price and the concept of rate structure premium—a measure of the difference between actual expenditure and the cost of consumption priced at marginal cost.

More recent studies have attempted to test price specification empirically. Billings and Agthe (1980) argue that marginal price and income adjustment are correct in the context of water demand. Griffin and Martin (1981) find fault with their analysis as the endogeneity of price is not corrected. Foster and Beattie (1981) additionally argue that the distinction between average and marginal costs is inconsequential empirically.² Henson (1984) and Dubin (1985) confirm the endogeneity of measured marginal price using specification tests which compare consistent instrumental variables estimates with ordinary least squares (Haussman (1978)). However, the power of these results rests on the degree to which exogenously forecast rates correspond to endogenously selected rates.³ Furthermore, consistent estimation methods employed to date have failed to provide a practical way of comparing alternative rate structures in their impact on demand.

In Section 2, we develop an empirical model of electricity demand subject to a quantity determined rate schedule and suggest a new procedure to estimate population test variation. Section 3 presents the results using cross-sectional micro-level data from the 1975 Washington Center for Metropolitan Studies (WCMS) survey and provides evidence on the prevalence of block switching. A final section provides a summary and some conclusions.

II. Empirical Model of Demand Subject to Endogenous Rates

In this section we follow recent empirical studies which indicate the importance of cross-sectional taste variation. In studies of labor supply, for example, small R-squareds are symptomatic
of the inadequacies of explaining observed hours of work under the assumption of a representative individual. Population taste variation for electricity demand is expected to be particularly important when considering alternative rate structures aimed at benefiting low income households.

Following Burtless and Hausman (1978) and Hausman (1983), we assume the income effect to be randomly distributed across individuals. To compare budget segments, an indirect utility function, \( V[p, y; \beta] \), in prices, income, an unknown income parameter \( \beta \), is found which is consistent with the assumed demand specification. Each value of \( \beta \) between zero and \( + \beta \) gives rise to an associated global optimum level of electricity demand given a particular rate structure.

The probability that desired demand lies in a given range is then equivalent to the probability of an associated range in \( \beta \). Actual demand is assumed to differ from desired demand by an additive disturbance \( \eta \).

In the non-convex budget case consisting of two segments, Burtless and Hausman (1978) demonstrate that desired demand will fall in the steeper budget segment for \( 0 < \beta < \beta^* \) and will fall in the marginal segment for \( \beta^* < \beta \). The parameter \( \beta^* \) denotes a point in the parameter space of equal indirect utility for both segments \( V[p_1, y_1; \beta^*] = V[p_2, y_2; \beta^*] \). If \( F(\eta, \beta) \) denotes the cumulative distribution function for \( \eta \) and \( \beta \) and \( \zeta_j \) denotes desired demand in the \( j^\text{th} \) segment \( (j = 1 \text{ or } 2) \) then the likelihood of observed demand \( q \) is

\[
\int_0^{\beta^*} \int_\eta (q - \zeta_1) dF(\eta, \beta) + \int_{\beta^*}^{+\beta} \int_\eta (q - \zeta_2) dF(\eta, \beta)
\]

(1)

The full-information maximum likelihood solution to convex and non-convex budget set estimation is implementable provided simple distributional assumptions are made concerning \( \eta \) and \( \beta \). If more than one or two parameters are assumed to vary in the population, the requirement of evaluating multiple integrals over non-rectangular regions implies too complex a problem for maximum likelihood.

As an alternative to the maximum likelihood solution we consider consistent estimation of moments of the random parameter distribution. Write the demand equation as:

\[
y = \sum_{j=1}^J a_j X_j + Z\delta + \varepsilon
\]

(2)

where \( a_j \) is the random coefficient of variable \( X_j \), \( \delta \) is a column vector of non-random parameters corresponding to the variables \( Z \), and \( \varepsilon \) is an additive disturbance. We assume that \( a_j = a_j + v_j \) with \( E[v_j] = 0 \) and \( E[v_j^2] = a_j^2 \) and that \( E[\varepsilon] = 0 \), \( E[\varepsilon^2] = \sigma^2 \). Under the maintained assumption that the \( X_j \) and \( Z \) are uncorrelated asymptotically with the disturbances \( \varepsilon \) and \( v_j \), we could proceed to estimate the parameters \( a_j \), \( \delta \), and \( a_j^2 \), \( \sigma^2 \) using the methods of Hildreth and Hauck (1968). The Hildreth-Hauck procedures is not applicable in the presence of stochastic regressors which has been demonstrated to be an important consideration in the specification of demand subject to endogenous rates (Dubin (1985)).
Fortunately, a simple extension of the Hildreth-Hauck method exists which does guarantee consistency in the presence of stochastic regressors. Note that equation (2) implies:

\[ y = \sum_{j=1}^{J} \bar{a}_j x_j + \bar{Z} \theta + \bar{\eta} \quad \text{with} \quad \bar{\eta} = \bar{\epsilon} + \sum_{j=1}^{J} \bar{\sigma}_j x_j. \]

(3)

As \( E(\bar{\eta}) = 0 \) and \( E(\bar{\eta}^2) = \bar{\sigma}^2 + \sum_{j=1}^{J} \bar{\sigma}_j^2 \bar{x}_j^2 \), a two-step moment estimation procedure is suggested. In the first step, instrumental variable estimation of (3) provides consistent estimates of \( \bar{a}_j \) and \( \bar{\theta} \). In the second step, consistent estimates of the variances \( \bar{\sigma}^2 \) and \( \bar{\sigma}_j^2 \) are obtained through an auxiliary regression of the squared residuals on a constant term and the variables \( \bar{x}_j^2 \). While consistency of the parameter estimates given in the second-stage auxiliary regression is guaranteed, standard errors will be incorrect.

We implement this procedure using a standard linear specification for electricity demand: \( Q = \alpha + \beta (\text{income}) + Z \theta + \epsilon \) where \( \beta = \bar{\beta} + \delta \) denotes the random income parameter and where \( Z \) represents a vector of explanatory variables for the household appliance stock and other socio-economic effects. The form of this model and the estimation procedure (instrumental variables) are based on Dublin (1985) using cross-sectional micro-level data from the 1975 Washington Center for Metropolitan Studies (WCOM) survey.\(^6\) Dublin concluded that (1) measured average price and measured marginal price are statistically endogenous so that least squares techniques are not appropriate for the determination of price elasticities, (2) the statistical contribution of the rate structure premium adjustment is negligible and (3) consumer behavior in the demand for electricity follows the marginal rather than average price specification.\(^7\) For brevity we include summary statistics for the selected model in Table 1 and the instrumental variables estimates in Table 2.

### Table 1

**VARIABLE MEANS FOR ELECTRICITY DEMAND MODEL**

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Description</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>AKW75</td>
<td>monthly consumption of electricity in 1975</td>
<td>916.5</td>
</tr>
<tr>
<td>RATE</td>
<td>measured marginal price in 1975</td>
<td>0.02427</td>
</tr>
<tr>
<td>INCOME</td>
<td>monthly income of household head</td>
<td>1322</td>
</tr>
<tr>
<td>RSP</td>
<td>measured rate structure premium</td>
<td>5.151</td>
</tr>
<tr>
<td>WHE</td>
<td>electric water heat dummy</td>
<td>0.2728</td>
</tr>
<tr>
<td>SHE</td>
<td>electric space heat dummy</td>
<td>0.1411</td>
</tr>
<tr>
<td>ROOFS</td>
<td>number of rooms in household</td>
<td>6.078</td>
</tr>
<tr>
<td>PERSONS</td>
<td>number of persons in household</td>
<td>3.550</td>
</tr>
<tr>
<td>CAC</td>
<td>central air-conditioning dummy</td>
<td>0.2890</td>
</tr>
<tr>
<td>CDDC</td>
<td>(annual cooling degree days) * (CAC)</td>
<td>463.7</td>
</tr>
<tr>
<td>RACHNUM</td>
<td>number of room air-conditioners</td>
<td>0.4382</td>
</tr>
<tr>
<td>CDDRACHNUM</td>
<td>(annual cooling days) * (RACHNUM)</td>
<td>642.3</td>
</tr>
<tr>
<td>AUTOWASH</td>
<td>automatic washing machine dummy</td>
<td>0.8898</td>
</tr>
<tr>
<td>AUTODSH</td>
<td>automatic dishwasher dummy</td>
<td>0.4921</td>
</tr>
<tr>
<td>FOODFRZ</td>
<td>food freezer dummy</td>
<td>0.5323</td>
</tr>
<tr>
<td>ELECTRANGE</td>
<td>electric range dummy</td>
<td>0.6411</td>
</tr>
<tr>
<td>ECOLTHDR</td>
<td>electric clothes dryer dummy</td>
<td>0.4990</td>
</tr>
<tr>
<td>TVM</td>
<td>black and white television dummy</td>
<td>0.5806</td>
</tr>
<tr>
<td>CITV</td>
<td>color television dummy</td>
<td>0.7446</td>
</tr>
</tbody>
</table>

\(a\) A subsample of the WCOM survey was selected so that complete information was available for each individual. Details may be found in Dublin (1985).
TABLE 2
ELECTRICITY DEMAND MODEL

<table>
<thead>
<tr>
<th>Variablea</th>
<th>Instrumental Variable Estimatesb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Marginal Price (RATE)</td>
<td>-6006 (-3.269)</td>
</tr>
<tr>
<td>Income</td>
<td>.07570 (3.071)</td>
</tr>
<tr>
<td>WHE</td>
<td>404.5 (10.15)</td>
</tr>
<tr>
<td>SHE</td>
<td>714.9 (14.40)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.7051</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>744</td>
</tr>
<tr>
<td>Sum of Squared Residuals</td>
<td>.9166e+8</td>
</tr>
<tr>
<td>Standard Error of Regression</td>
<td>355.6</td>
</tr>
</tbody>
</table>

a In Table 2 coefficient estimates are not reported for the variables: PERSONS, BWTV, ROOMS, MELCACA, CDDCACA, CAC, RACNUM, CDDRACNUM, FOODFRE, ElectrNGE, CLRTV, ECLTHDR, AUTODSH, AUTOWSH, and the intercept. The dependent variable is AKWH73.
b t-statistics presented in parentheses.

Results of the auxiliary regression which determine the moments $\sigma^2$ and $\sigma^2_j$ indicate that

$$e_t^2 = (0.8447e+5) + (0.01762)(\text{Income}_t^2) + \tilde{\xi}_t^2$$

(4)

where $\tilde{\xi}_t$ is the instrumental variable estimated residual and $e_t$ denotes the fitted residual in the auxiliary regression. Adjusting for degrees of freedom we find that

$$\frac{1}{(744-19)} \sum_{t=1}^{744} e_t^2 = (0.8447e+5)(744)/(744-19) + (0.01762)(744)/(744-19)$$

(5)

The left-hand side of (5) is a large-sample estimate of variance associated with the unobservable in the demand equation. Using the regression standard error from Table 2, (355.6) and the population mean and variance for Income, (1322.0 and 0.4508e+6, respectively) we find that $\text{var}(e) = (0.866837e+5)$ and $\text{var}(\beta \cdot \text{Income}) = (0.39753e+5)$. Thus, over 45 percent of total variance is accounted for by randomness in the income taste parameter. Hausman (1983), by way of contrast, reports that virtually all unexplained variation in labor supply may be attributed to taste variation.

At sample means the instrumental variable estimates imply a price elasticity of -0.159. Taylor (1975) reports both short-run and long-run price and income elasticities. Of nine estimates of residential elasticities two used marginal price. Each of the studies by Houthakker (1951a, 1951b) reports short-run elasticities of approximately -0.90. The instrumental variable estimates are well below this estimate in magnitude but are entirely consistent with other estimates of electricity demand price elasticity using an average price specification.8
III. Simulation of Demand Under Alternative Rate Structures

We now consider the problem of determining demand response subject to non-marginal changes in the underlying budget constraint set. To illustrate the method we consider two hypothetical experiments. In the first experiment, consumers choose between their observed block and a hypothetical block consisting of a flat rate set at the intra-marginal average price. In the second experiment, we illustrate the effect of a uniform increase of 30 percent in the observed lower block boundary.

For the purpose of making explicit probability statements we assume that \( \log \beta \) is a normal random variable with mean \( \mu \) and variance \( \lambda^2 \). The log-normal distributional assumption gives \( \beta \) positive support which is consistent with the assumption that electricity is a normal good. Moreover, the translation between the observed moments \((\bar{\beta}, \sigma_{\beta}^2)\) and the moments \((\mu, \lambda^2)\) is accomplished in a simple calculation.  

Recall that the log-normal distribution for \( \beta \) implies

\[
\bar{\beta} = \exp(\mu + \frac{1}{2} \lambda^2) \quad \text{and} \quad \sigma_{\beta}^2 = \exp(\lambda^2) - 1 \exp(2\mu + \lambda^2).
\]

From these we find:

\[
\lambda^2 = \ln(\sigma_{\beta}^2/\bar{\beta}^2 + 1) \quad \text{and} \quad \mu = \ln(\bar{\beta}^2/(\sigma_{\beta}^2 + \bar{\beta}^2)^{1/2}).
\]

From Table 2, \( \bar{\beta} = 0.0757 \) and from equation (4), \( \sigma_{\beta}^2 = 0.01762 \) which implies \( \mu = -3.2834 \) and \( \lambda^2 = 1.4048 \). The median of the \( \beta \) distribution occurs at \( \exp(\mu) = 0.0375 \) and indicates left skewness relative to the mean of 0.0757. A simple calculation reveals that 71 percent of the \( \beta \) distribution lies below its mean value.

Having estimated the random parameter distribution, we turn to a probabilistic comparison of alternative rate structures. Dubin and McFadden (1984) show that the indirect utility function

\[
V(p, y; \beta) = -\beta p [y_1 + (a/\beta)p + a/\beta^2 + Z\beta] \tag{6}
\]

is consistent with the demand equation

\[
x_1 = Q(p, y; \beta) = \frac{\partial V/\partial p}{\partial V/\partial y} = \beta y_1 + ap_1 + Z\beta. \tag{7}
\]

The coefficients and explanatory variables in (7) are selected to be consistent with the specification of desired demand given in Table 2. Price \( p_1 \) and income \( y_1 \) are defined appropriately for each budget segment, i.e., \( p_1 \) is the marginal price for segment 1 and \( y_1 \) is income less the rate structure premium adjustment in segment 1.

Equations (6) and (7) may be combined to express indirect utility as a function of observed demand:

\[
V(p_1, y_1; \beta) = e^{-\beta p_1 [x_1/\beta + a/\beta^2]} \tag{8}
\]

The distribution of indirect utility induced by \( \beta \) is not readily calculated given the form of equation (8). To find this distribution, we approximate (8) by a Taylor’s series expansion in \( \beta \) around its mean \( \bar{\beta} \):

\[
V(p_1, y_1; \beta) \approx V(p_1, y_1; \bar{\beta}) + \frac{\partial V(p_1, y_1; \bar{\beta})}{\partial \beta} (\beta - \bar{\beta}) \tag{9}
\]

with slope \( \partial V/\partial \beta \) given by:
\[ \frac{\partial V[p_1, y_i; \beta]}{\partial \beta} = e^{-\beta p_1 ((\beta y_i + X_1)/\beta^2 - 2p_1^3)} e^{-\beta p_1 [X_1/\beta + a/\beta^2]} (-p_1) \]
\[ = (e^{-\beta p_1}) [p_1^3(y_1 - p_1 X_1) - \beta (X_1 + p_1 a) - 2a] \]  

(10)

For concreteness we proceed with an analysis of the declining two-part rate structure. These assumptions are not restrictive and the method generalizes. The declining two-part tariff assumes a price \( \pi_1 \) for consumption up to and including an amount \( X \) and a price \( \pi_2 \) for any additional consumption. We first calculate the probability that the indirect utility associated with the first budget segment is greater than the indirect utility associated with the second budget segment taking into account the possibility that certain ranges of prices make either budget segment infeasible. Given a declining tariff \( \pi_2 < \pi_1 \), we find:

\[ \Pr(V_1 \geq V_2) = \Pr(V_1 \geq V_2 | \pi_2 < \pi^* \leq \pi_1) \cdot \Pr(\pi^* < \pi \leq \pi_1) \]
\[ + \Pr(V_1 \geq V_2 | \pi^* < \pi_2) \cdot \Pr(\pi^* < \pi_2) \]
\[ + \Pr(V_1 \geq V_2 | \pi^* > \pi_1) \cdot \Pr(\pi^* > \pi_1) \]  

(11)

where \( V_1 = V(p_1, y_i; \beta) \) and \( p_1 = \pi_1 \), \( y_1 = y - \text{income}, p_2 = \pi_2, \) and \( y_2 = y - (\pi_1 - \pi_2)X \). We define the boundary price \( \pi^* \) as the implicit solution to \( Q[\pi^*, y_i; \beta] = X \). Note that this price is itself random given a distribution for \( \beta \).

The first term in equation (11) is simply the joint probability \( \Pr(V_1 \geq V_2 \text{ and } \pi^* \leq \pi_1) \). In the second term, the condition \( \pi^* < \pi_2 \) implies \( V_1 \geq V_2 \) with certainty since the second segment is necessarily infeasible. In the third term, \( \Pr(V_1 \geq V_2) \) is zero as the first segment may not be selected when \( \pi^* \) is strictly larger than \( \pi_1 \). It follows that:

\[ \Pr(V_1 \geq V_2) = \Pr(V_1 \geq V_2 \text{ and } \pi_2 < \pi^* \leq \pi_1) + \Pr(\pi^* < \pi_2) \]  

(12)

Using the Taylor's series expansion in equation (9) we find:

\[ \Pr(V_1 \geq V_2 \text{ and } \pi_2 < \pi^* \leq \pi_1) \]
\[ = \Pr((m_2 - m_1)(\beta - \beta^0) \leq (u_1 - u_2)) \text{ and } \pi_2 < \frac{X - \beta y - 2\delta}{\alpha} \leq \pi_1 \]
\[ = \Pr((m_2 - m_1)(\beta - \beta^0) \leq (u_1 - u_2)) \text{ and } \frac{X - \beta}{Y} < \beta - \beta^0 \leq \frac{X - X_1}{Y} \]  

(13)

where we define \( \beta^0 = \alpha p_1 + \beta y_1 + 2\delta, m_1 = \partial V[p_1, y_i; \beta]/\partial \beta, \) and \( u_1 = V(p_1, y_i; \beta) \). Similarly, it can be shown that:

\[ \Pr(\pi^* \leq \pi_2) = \Pr(\frac{X - X + 2\delta}{\alpha} \leq \pi_2) \text{ and } \Pr(\beta - \beta^0 \leq \frac{X - X_2}{Y}) \]  

(14)

Collecting (12), (13), and (14) we find:

\[ \Pr(V_1 \geq V_2) = \Pr(\frac{X - X_1}{Y} \leq \beta - \beta^0 \leq \min(\frac{X - X_1}{Y}, \frac{u_1 - u_2}{m_2 - m_1})) \]
\[ + \Pr(\beta - \beta^0 \leq \frac{X - X_2}{Y}) \text{ when } m_2 > m_1 \]  

(15)
Experiment 2: In the second experiment, we consider the effect of a uniform increase of 30 percent in the observed lower block boundary for each household. In this case we expect that a greater number of households will find that the first budget segment yields greater utility. Repeating the probability calculation we find a population average value of 0.3382 for \( \text{Prob}[V_1 \geq V_2] \) with a standard deviation of 0.5518. The 30 percent increase in lower block boundary therefore induces a 16.94 percent increase in the likelihood that an individual will select the intra-marginal price over the observed marginal rate. While it is true that individual predicted probabilities will differ from the sample average values, the magnitude of the difference in population means is large enough to indicate a systematic shift.

IV. Summary and Conclusions

In the introduction we asked whether the instrumental variables estimation correction to price endogeneity is likely to be of much value and whether block switching is a likely result of changes in the prices of non-marginal blocks. Experiment 1 has demonstrated that the observed block is in fact closely related to the predicted optimal block so that we expect the instrumental variables estimation procedures to work quite well. Experiment 2 has demonstrated that block switching is a probable outcome of rate changes. On the way to answering these questions we have demonstrated a practical way of comparing alternative rate structures in their impact on demand. The approach suggested a simple mechanism for
recovering the variance components of demand from which we concluded that individual taste variation is quite important in explaining the distribution of observed demands in cross-section data. The methods presented in this paper should be useful in estimating the distributional impacts on consumers as rates are shifted from declining to inverted block form.

FOOTNOTES

1. Consistent parameter estimates may be obtained by an instrumental variables procedure due to McFadden, Kirshner, and Puig (1978). Modifications of this basic approach have been employed by Hausman, Kinnucan, and McFadden (1979), Barnes, Gillingham, and Hageman (1981), Henson (1984), and Dubin (1985).

2. Foster and Beattie's empirical evidence is suspect, however, as they fail to correctly implement the test for equality of regressors in two regressions and fail to allow for price endogeneity.

3. The instrumental variable technique utilizes predicted rather than actual consumption to determine measured marginal price. In forming predicted consumption levels, all endogenous variables are purged from the set of explanatory variables. One must insure that the instruments so constructed are not exact linear combinations of the exogenous variables included in the electricity demand equation. This is usually not a problem given the non-linearity of the rate schedule and given the existence of other prices which are exogenous. Dubin (1985) uses the tail-end block price in exactly this role.
4. The objective of these studies was the determination of labor supply with non-linear net wages. Hall (1973) had demonstrated the usefulness of the net wage approach and defined the concept of virtual income which is identical to our definition of income less rate structure premium.

5. Terza and Welch (1982) pursue the suggestion of Burtless and Hausman (1978) and view the choice of segment in nonconvex budgets sets as a censored sample problem. Terza and Welch develop the selectivity corrections necessary to consistently estimate demand when taste parameters are invariant in the population. Our approach differs in that we allow taste parameters to vary in the population and pursue a moment estimator to avoid the complexities of maximum likelihood.

6. The WCRS data is well suited to the analysis at hand. It is one of few data sets for which the explicit matching of billing schedules to households has been possible.

7. A source of bias not discussed in this paper arises from the endogeneity of appliance ownership dummies. Generally, unobserved factors which influence the choice of a durable will also influence its use. For a complete discussion of this problem see Dubin and McFadden (1984) who find evidence that this leads to under estimates (in magnitude) of the true price effects.

8. Studies by Acton, Mitchell, and Mowill (1976) and Taylor, Blattenberger, and Verleger (1977), find short-run price elasticities from -.08 to -.35 with endogenous marginal price specifications.

9. An alternative assumption of the truncated normal distribution for β would not allow this simple translation and would be equally arbitrary.
REFERENCES


