Firing in Non-Repeated Incentive Contracts

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Abstract

When the presence of limited liability restricts a principal from imposing monetary fines on an agent in case of poor performance, the principal might use other kinds of punishment threats to deter the agent from shirking. If firing is costly to the agent it can be used by the principal even in non-repeated contracts. This paper considers the conditions under which a profit-sharing arrangement combined with a certain firing rule improves the principal’s position compared to the situation in which firing is not an option. The optimal firing rule is established, and its effectiveness is considered as a function of exogenous economic variables. Possible applications of the proposed contract are suggested.
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1 Introduction

The phenomenon of "asset stripping" by current managers of state-owned enterprises in modern transition economies presents one of the more prominent examples of moral hazard. With the state unable to monitor managers' behavior and with uncertainty about their future, managers try to sell off the enterprises' resources for their own profit. Since immediate privatization of all state-owned enterprises, which would probably resolve this problem, is theoretically questionable and practically unrealizable, one must search for alternative ways to deter managers from shirking.

The traditional ways to resolve moral hazard problems are (i) monitoring of the agent's actions if it is not too costly; (ii) risk-sharing or profit-sharing arrangements where the agent can be fined or rewarded on the basis of observed performance; (iii) the threat of termination of the contract in case of poor performance in repeated relationships. In transition economies, however, monitoring of managers is likely to be impossible and imposing fines might not be feasible because of the presence of limited liability. Further, the threat of termination of employment is present regardless of manager's performance: once the enterprise is privatized the new owner will probably change the management.

Other similar problems exist in transition economies. Because the environment is rapidly changing agents will have many short-term partners or subcontractors who may not be fully trustworthy, because of the one-shot nature of the proposed deals. Limited

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1For more details on "asset stripping" see Lipton and Sachs (1990), Vickers and Yarrow (1991) and Frydman and Rapaczynski (1991).
liability is also a problem because of a poor legal system that is not able to enforce monetary punishments even in cases of obvious fraud.

Similar problems arise in fully functioning market economies. In a procurement process the government is often restricted in the amount of fines it can assess a contractor for a low quality instrument; by its nature the process of building a new instrument is highly uncertain in its costs and output characteristics; and because of uniqueness of the task the same contract is almost never repeated. Finally, we can think of a manager (or any employee) who works on a profit-sharing schedule with limited liability and is on the verge of retiring. As in any finitely repeated relationship, we can expect more shirking on his part in the last employment period since he does not have to care about the future employment.

The common features of the above situations with moral hazard are the one-shot nature of the relationship, the presence of limited liability and a very high cost of monitoring for the principal. In what follows, we design a contract between a principal and an agent that will, under given circumstances, help to prevent the latter from shirking.

In his analysis of transition economies Tirole (1991) argued that “low-powered” managerial incentive schemes, such as career concerns, should be used during the first, “noisy” phase of transition, while “high-powered” incentives, including profit-sharing, are appropriate for later phases. Keeping in mind that the “noisy” environment is rather typical for the examples presented above, we propose a contract that combines a profit-sharing arrangement with a firing rule, where the agent may be fired depending on his performance. In this case, the agent will suffer the personal costs associated, for example, with a reputation for failing at his task and being suspected of shirking. If the agent succeeds in his performance and is not fired, he is then entitled to a share of the output (or profit) after the project is carried out.

In application to the managerial control example, the proposed scenario means that a current manager of a state-owned enterprise, if he does not get fired during the transition period, might be rewarded with a share of revenue (or, alternatively, a share of the enterprise’s value in stock options) once the enterprise is privatized. Presumably, the less resources the manager steals and the better he prepares the enterprise for market conditions, the higher the enterprise’s future expected profit is going to be, and the higher the price it will be sold for to the new owner. In case of poor performance, though, the manager may be fired (or jailed) and suffer personal costs associated with future career concerns.

For one-shot deals, temporary partners may use not only profit-sharing but also may punish each other with the reputation of a cheater when they observe the results of each other’s performance. If the agent’s reputation becomes well-known in the business world,

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2In what follows we will refer to the principal as “she” and to the agent as “he”.


its effect might be significant to him. Similarly for the procurement process, the proposed contract means that the contractor should be rewarded on the basis of the quality of the instrument that he builds; and, besides, poor instrument quality may produce bad reputation for the contractor, which might affect his future contracting opportunities - even if he does not contract with the same principal. Finally, an employee near retirement might care whether he retires "with honor" or with a questionable reputation as a dishonest worker.

The main question we ask in the paper is whether a profit-sharing contract combined with a firing rule has any advantages for the principal over a contract where firing is not an option. We consider what type of firing rule the principal should choose, whether the firing threat will induce the agent to shirk less and whether the agent can be paid a lower share when he is paid compared to the solution to the original moral hazard problem without firing. Further, we consider the sensitivity of such arrangements to the level of firing cost that the agent faces and to the total amount of resources available for investment.

The moral hazard problem has been extensively studied in the economic literature, starting with Holmstrom (1979) and Shavell (1979), who analyzed the optimal risk-sharing rule when the principal cannot observe the agent's actions and the production process includes random shocks. Sappington (1983) considered limited liability risk-sharing contracts, when there are ex-post limitations on the penalty imposed on the agent, for the case when both principal and agent are risk-neutral. Literature where the firing possibility is explicitly incorporated into the risk-sharing contract is scarce. Chwe (1990) considered a two-state model where both monetary rewards and non-monetary punishments are included into the incentive scheme. He found that if rewards and punishments are both costly to the principal, it is never optimal to use them together. Zou (1991) also studied threat-based incentive mechanisms under moral hazard and adverse selection, where the agent could self-select a target-penalty level. In his setup the agent received different but fixed payments depending on whether he has been able to achieve the target level.

Some papers consider the effects of various kinds of "firing-type" threats on an agent's behavior. Grossman and Hart (1982) investigated the incentive effects of the threat of bankruptcy or takeover bids on the management of a widely held corporation. They found that in general such threats lead the managers to less shirking and to the achievement of higher profits.

A number of papers (Stiglitz and Weiss (1983), Bolton and Scharfstein (1990)) used repeated agency models to explore whether and when the threat of the termination of the contract in the first periods solves the moral hazard problem. Finally, similarities with firing rules are common in the electoral dynamics literature. In this context, a representative voter's decision to reelect a politician is similar to making a choice between
renewing the contract with the previous agent-politician as opposed to employing another one. For example, Ferejohn (1986), Austen-Smith and Banks (1989) and Banks and Sundaram (1991) present models where an incumbent's performance in office influences the probability of being reelected for next period; voters set this kind of reelection rule in an attempt to induce politicians to work harder. However, these "contracts" lack risk-sharing elements and are mostly studied from a dynamic prospective where the voter may learn more about the executive from period to period, whereas we consider a one-period model.

Although suggestive, the moral hazard and retrospective voting literatures do not adequately model the specific problem with which we are concerned. To do so in this paper we combine, explicitly, a firing rule and a profit-sharing arrangement — two tools previously used mostly separately to solve the agency problem.

2 The model

To study the type of arrangement presented in Section 1, we consider the following model. At the beginning of the period the principal pays the agent a fixed salary $W$ and provides him with an amount $R$ of the resources which the agent is supposed to invest into task implementation (some kind of production) specified by the principal. Since the agent's investment decision is unobservable to the principal, the agent may choose to use a part of the available resources in his own interest and invest only the remaining part $I$, $0 \leq I \leq R$. Let the outcome $Q$ of the production process be a non-negative valued stochastically increasing function of $I$:

$$Q = Q(I, \varepsilon) \geq 0 \text{ for any } I, \varepsilon$$

$$I \in [0; R]$$

$$\varepsilon \in (-\infty; +\infty)$$

where $\varepsilon$ is a random shock distributed according to the cumulative distribution function $H(\varepsilon)$. Equivalently, we can use a parameterized distribution function $F(Q; I)$ and its

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3 We include the fixed part $W$ into the agent's salary to consider whether it affects the solution to our problem.

4 We keep both $W$ and $R$ exogenous in order to concentrate on the effects of the firing threat on the properties of the contract.

5 Thus in our setup the agent's choice variable is investment level as opposed to traditionally used effort level; this is done to make the model closer to the examples presented in the introduction. Nevertheless in what follows we keep up with the standard assumptions of moral hazard literature, such as separability of agent's utility in privately used resources and monetary rewards.

6 Assumption $Q \geq 0$ means that we do not consider a possibility of losses from production; the assumption does not affect any of our results.

7 For the parameterized distribution formulation, see, for example, Hart and Holmstrom (1987), p. 78.
density \( f(Q; I) \) to denote the distribution of \( Q \) given \( I \). The following assumptions are made about the parameterized distribution function:

1. \( F(Q; I) \) is continuous and three times differentiable;
2. \( \lim_{Q \to \infty} f(Q) = \lim_{Q \to \infty} f'(Q) = 0; \)
3. \( F_I(Q; I) < 0 \): that is, the distribution changes with \( I \) in the sense of first order stochastic dominance. Thus with higher investment the agent can decrease the probability of a low outcome;
4. \( f_I(Q; I)/f(Q; I) \) is increasing in \( Q \). That is, the Monotone Likelihood property (Holmstrom (1979) and Shavell (1979)) holds;
5. For any \( I \in [0, R] \) there exists a point \( \hat{Q}(I) \) such that \( f_I(Q; I) = 0; \)
6. \( \hat{Q}(I = 0) = 0. \)

Assumptions 1-4 on the distribution are standard for the moral hazard literature and we do not need to comment on them; assumptions 5 and 6 are not common but it is easy to see that they hold for a quite large family of parameterized distribution functions. For example, these assumptions are true for the class of distribution functions which are obtained by shifting the distribution of \( \varepsilon \) by some amount dependent on \( I \) without leaving its other characteristics unchanged, given that the distribution density of \( \varepsilon \) has at least one extremum\(^8\). In this special case \( \hat{Q}(I) \) corresponds to the extremum of the parameterized distribution density function; therefore for the sake of simplicity we will often refer to \( \hat{Q}(I) \) as to the “peak” of the distribution, keeping in mind inaccuracy of the term.

After the production takes place and its outcome \( Q \) becomes observable to everybody, the agent is entitled to receive a share of the output \( \alpha(Q) \), \( 0 \leq \alpha(Q) \leq Q \), prespecified by

\[ Q = Q(I, \varepsilon) = \begin{cases} r(I) + \varepsilon & \text{if } r(I) + \varepsilon \geq 0 \\ 0 & \text{otherwise} \end{cases}, \varepsilon \in (-\infty; +\infty), \]

where \( r(I) \) is increasing non-negative valued deterministic function, \( r(0) = 0 \), and \( \varepsilon \) is a random shock distributed according to the cumulative distribution function \( H(\varepsilon) \), with \( H''(0) = 0 \), i.e., the density function has a peak at \( \varepsilon = 0 \). (Truncation at \( Q = 0 \) is used to insure that \( Q \) is non-negative and does not affect any of our conclusions.) Then \( F(Q; I) = H(Q - r(I)) \), and assumptions 3, 5 and 6 follow: (3) \( F_I(Q; I) = -H'(Q - r(I)) * r'(I) < 0 \) because \( r'(I) > 0 \) by assumption and \( H''(Q - r(I)) > 0 \) since it is a density function. (5) \( f(Q; I) = H'(Q - r(I)) = 0 \) at \( Q - r(I) = 0 \) by assumptions and hence \( \hat{Q} = r(I) \) is a peak of \( f(Q; I) \). Next, \( f_I(Q; I) = -r'(I) * H''(Q - r(I)) = -r'(I) * f'(Q; I) \), and \( \hat{Q} = r(I) \) being a peak implies that \( f'_I(Q; I) = H''(r(I) - r(I)) = 0 \), and consequently \( f_I(Q; I) = 0 \). (6) \( r(0) = 0 \) implies \( \hat{Q}(I = 0) = r(I) = 0. \)

\(^8\)To see that this statement is true, consider \( Q(I, \varepsilon) \) of the form:
the principal (for simplicity suppose both $Q$ and $\alpha(Q)$ are expressed in monetary terms). The rest of the output $Q - \alpha(Q)$ goes to the principal. Besides, depending on how badly the agent fails with production process, he might get "fired" (in a sense explained in Section 1) according to the firing rule $\Phi(Q)$, also predetermined by the principal. Without loss of generality we can assume that the rule $\Phi(Q)$ specifies the probability of firing the agent as a function of his accomplishment $Q$. In case the agent is fired he bears a personal cost $C$, although he may still get some share of output $\alpha(Q)$.

The agent chooses the level of investment $I$ to maximize his expected utility $U$; clearly, the dilemma he faces is between having more immediate utility from stealing the resources today versus investing more to increase the probability of not being fired and of getting a higher share of the possibly higher output tomorrow. Assume that the agent's utility is separable in resources and income and let $g(R - I)$ be the agent's utility from stealing $R - I$ resources, and $G(M)$ - his utility for money. Given that the agent is paid a fixed salary $W$, expects to get a share of output $\alpha(Q)$ and bears the private cost $C$ if he is fired, the income that the agent expects to get is $M = W + \alpha(Q) - C$ if he is fired and $M = W + \alpha(Q)$ otherwise. Assume that both $g(x)$ and $G(x)$ are positive valued for $x > 0$, negative-valued for $x < 0$, twice differentiable, increasing in $x$ and concave functions, with $g(0) = G(0) = 0$, $g'(0) = \infty$, $g'(R) = 0$ and $0 < g'(x) < \infty$ for $x \in (0, R)$. The agent's objective function then is:

\[ U(I) = g(R - I) + E[G(W + \alpha(Q) - C) \ast \Phi(Q)] + E[G(W + \alpha(Q)) \ast (1 - \Phi(Q))] = \]
\[ = g(R - I) + \int [G(W + \alpha(Q) - C) \ast \Phi(Q) + G(W + \alpha(Q)) \ast (1 - \Phi(Q))] f(Q; I) dQ \]

where $E(\cdot)$ denotes the expectation. Thus the agent's problem can be written as:

\[
\max_I \quad g(R - I) + \int [G(W + \alpha(Q) - C) \ast \Phi(Q) + G(W + \alpha(Q)) \ast (1 - \Phi(Q))] f(Q; I) dQ \\
0 \leq I \leq R
\]

The principal has to determine the firing rule $\Phi(Q)$ and the payment schedule $\alpha(Q)$ to maximize her expected utility. In our interpretation the payment schedule can not be used to fine the agent for low output so we impose the restriction $\alpha \geq 0$; $\Phi(Q)$ is the probability of firing which implies $0 \leq \Phi(Q) \leq 1$. Let the principal benefits from the share of output she gets according to the utility function $V(Q - \alpha(Q))$, where $V(x)$ is

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9Alternatively, we could require that the agent cannot get any share of output in case he is fired; however, as we show in Section 3 below, this does not change the solution to the problem: it is never optimal to pay any positive share to the fired agent.

10This assumption is standard in the principal-agent literature. See, for example, Laffont(1989).

11Alternatively, we could make the agent's utility also separable in the part regarding disutility of being fired. However this would not change any of our findings but would instead violate traditional utility representation of the moral hazard models.
increasing in $x$, concave and twice differentiable. The principal’s objective function is then

$$S(\alpha(Q)) = \int V(Q - \alpha(Q)) f(Q; I) dQ$$

We can now rewrite the problem in standard principal-agent form using the first-order approach:

$$\max_{\Phi(Q), \alpha(Q), I} \int V(Q - \alpha(Q)) f(Q; I) dQ$$

subject to:

$$g(R - I) +$$

$$+ \int [G(W + \alpha(Q) - C) * \Phi(Q) + G(W + \alpha(Q)) * (1 - \Phi(Q))] f(Q; I) dQ \geq U^o$$

$$-g'(R - I) +$$

$$+ \int [G(W + \alpha(Q) - C) * \Phi(Q) + G(W + \alpha(Q)) * (1 - \Phi(Q))] f_I(Q; I) dQ = 0$$

$$\alpha(Q) \geq 0 \text{ for every } Q$$

$$0 \leq \Phi(Q) \leq 1 \text{ for every } Q$$

$$0 \leq I \leq R$$

Expression (1) is the principal’s objective function which she is maximizing subject to: the voluntary participation constraint (2), the incentive compatibility constraint (3), the “limited liability”\textsuperscript{13} constraint on the risk-sharing (4) and the constraints (5) and (6) on the domains of the choice variables $\Phi(Q)$ and $I$.

Expression (2) shows that the agent will be willing to participate in the contract when the expected utility of participation is not less than the utility level $U^o$ that he can get elsewhere. Note that in our setup this requirement is trivially satisfied when $G(W - C) \geq U^o$, because constraint (4) guarantees that $\alpha(Q)$ is non-negative and $g(x) \geq 0$ for any $x \geq 0$ by assumption, and $f(Q; I) > 0$ since it is a density function. In our analysis we will consider how the solution to the problem is different when the voluntary participation constraint is binding as opposed to the case when it is not.

Condition (3) requires that the agent’s choice of $I$ is the best available for him under this contract. As in standard models of this type, we use the first order conditions for the agent’s objective function to replace his maximization problem. In Section 4 below, we present sufficient conditions for this to be appropriate.

Expression (4), the limited liability constraint, is imposed because the agent cannot be fined for poor performance.

\textsuperscript{12}Since the amount of resources $R$ available to the agent is exogenously given, we do not count it as a cost to the principal.

\textsuperscript{13}We use this name as Sappington (1983) did.
Expression (5) is the constraint on the domain of \( \Phi(Q) \) imposed since \( \Phi(Q) \) denotes probability, and expression (6) is the constraint on the domain of \( I \).

Before turning to the solutions to the above problem, we first present an important property of the parameterized distribution function implied by our assumptions.

**Lemma 1** If a parameterized distribution function has a point \( \tilde{Q}(I) \) such that \( f_I(\tilde{Q}; I) = 0 \), the Monotone Likelihood Property implies that \( f_I(Q; I) < 0 \) for all \( Q < \tilde{Q} \) and \( f_I(Q; I) > 0 \) for all \( Q > \tilde{Q} \).

**Proof** The Monotone Likelihood property requires \( f_I(Q; I)/f(Q; I) \) to increase with \( Q \); \( f(Q; I) \) is always positive. Then if \( f_I(\tilde{Q}; I) = 0 \) for some \( \tilde{Q}(I) \), it follows that \( f_I(Q; I) < 0 \) for all \( Q < \tilde{Q} \) and \( f_I(Q; I) > 0 \) for all \( Q > \tilde{Q} \).

\[ \text{Q.E.D.} \]

In the next section we present the solution to the problem (1)-(5) ignoring the constraint (6) for the cases when the voluntary participation constraint is and is not binding and then show that the incentive compatibility constraint (3) indeed always guarantees that (6) can be ignored as long as \( W \leq C \).

### 3 Solutions

We now turn to the solutions. The Lagrangian for our problem with the voluntary participation constraint is:

\[
L(I, \alpha(Q), Q^*, \nu, \mu, \eta, \beta, \gamma) = \int \mathcal{V}(Q - \alpha(Q)) f(Q; I) dQ +
\]
\[
+ \nu \{ g(R - I) + \int [G(W + \alpha(Q) - C) \Phi(Q) +
+ G(W + \alpha(Q)) \Phi(Q)] f(Q; I) dQ - U^* \} +
\]
\[
+ \mu \{ -g'(R - I) + \int [G(W + \alpha(Q) - C) \Phi(Q) +
+ G(W + \alpha(Q)) (1 - \Phi(Q))] f_I(Q; I) dQ \} +
\]
\[
+ \eta \{ \alpha(Q) + \beta(Q) \Phi(Q) + \gamma(Q) \Phi(Q) \} ,
\]

where \( \nu, \mu, \eta(Q) \) and \( \beta(Q) \) and \( \gamma(Q) \) are the Lagrange multipliers corresponding to the constraints (2), (3), (4) and (5).

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The necessary (first order) conditions for optimal \( \alpha, I, \) and \( \Phi(Q) \) are:

\[
\frac{V'(Q - \alpha(Q))}{G'(W + \alpha(Q) - C) \Phi(Q) + G'(W + \alpha(Q)) (1 - \Phi(Q))} = \nu + \mu \frac{f_I(Q; I)}{f(Q; I)} +
\]

\[+ \frac{\eta(Q)}{[G'(W + \alpha(Q) - C) \Phi(Q) + G'(W + \alpha(Q)) (1 - \Phi(Q))]f(Q; I)} \quad \text{for all } Q (8)\]

\[
\int V(Q - \alpha(Q)) f_I(Q; I) dQ + \mu \{ g''(R - I) +
\]

\[+ \int [G(W + \alpha(Q) - C) \Phi(Q) + G(W + \alpha(Q)) (1 - \Phi(Q))] f_{II}(Q; I) dQ \} = 0 (9)\]

\[
[\nu f(Q; I) + \mu f_I(Q; I)] [G(W + \alpha(Q) - C) - G(W + \alpha(Q))] +
\]

\[+ \beta(Q) + \gamma(Q) = 0 \quad \text{for all } Q (10)\]

The conditions above provide a number of insights into the optimal firing rule, payment schedule and investment level. To find out how the introduction of firing changes the properties of the optimal contract, we start our analysis with consideration of the optimal firing rule \( \Phi(Q) \). In Proposition 1 below we present the conditions under which the introduction of a firing rule improves the solution of the problem for the principal, and determine the optimal firing rule. If we find that the firing threat is actually used, that is \( \Phi(Q) \neq 0 \) at least for some \( Q \), then the firing threat may be considered to be a useful regulation tool for the principal.

**Proposition 1** If there exists a positive cost \( C > 0 \) for the agent of being fired then the firing threat improves the solution of the problem for the principal; the optimal firing rule \( \Phi(Q) \) is given by

\[
\Phi(Q) = \begin{cases} 
1 & \text{for } Q < Q^* \\
0 & \text{for } Q \geq Q^*
\end{cases}
\]

where \( Q^* \) is the solution to the equation:

\[
\frac{f_I(Q^*; I)}{f(Q^*; I)} = -\frac{\nu}{\mu} .
\]

**Proof** First note that the firing rule is important only if it is costly for the agent. Indeed, if \( C = 0 \) then the agent’s utility function becomes:

\[
U = g(R - I) + \int [G(W + \alpha(Q)) \Phi(Q) + G(W + \alpha(Q)) (1 - \Phi(Q))] f(Q; I) dQ =
\]

\[= g(R - I) + \int G(W + \alpha(Q)) f(Q; I) dQ
\]

which does not depend on the firing rule \( \Phi(Q) \). The presence of any firing threat cannot affect the solution to the problem unless firing is costly for the agent, since otherwise firing does not affect either the principal’s or the agent’s payoff.
Now suppose that $C > 0$ and consider the necessary condition (10) for the optimal firing rule $\Phi(Q)$:

$$\left[\nu f(Q; I) + \mu f_i(Q; I)\right] * \left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] + \beta(Q) + \gamma(Q) = 0 \quad \text{for all } Q$$

Since $\beta(Q)$ and $\gamma(Q)$ are the Lagrange multipliers corresponding to the constraints $\Phi(Q) \geq 0$ and $\Phi(Q) \leq 1$, they are both equal to zero when these constraints are not binding. Thus for $Q \in \{Q|0 < \Phi(Q) < 1\}$ we have

$$\left[\nu f(Q; I) + \mu f_i(Q; I)\right] * \left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] = 0,$$

and since $G(\cdot)$ is strictly increasing the condition above is equivalent to

$$\frac{f_i(Q; I)}{f(Q; I)} = -\frac{\nu}{\mu} \quad \text{for all } Q \in \{Q|0 < \Phi(Q) < 1\} \quad (12)$$

Note however that since $\nu$ and $\mu$ are constants, the Monotone Likelihood Property implies that the solution to this equation is unique. Denote the $Q$ that solves (12) by $Q^*$; it follows that for all $Q \neq Q^*$ either $\Phi(Q) = 0$ or $\Phi(Q) = 1$ holds. The necessary condition for the optimal $\Phi(Q)$ becomes

$$\left[\nu f(Q; I) + \mu f_i(Q; I)\right] * \left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] + k(Q) = 0 \quad (13)$$

where $k(Q) = \beta(Q)$ for $Q \in \{Q|\Phi(Q) = 0\}$, $k(Q) = \gamma(Q)$ for $Q \in \{Q|\Phi(Q) = 1\}$, and $k(Q^*) = 0$. Next note that $\left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] < 0$ since $G(\cdot)$ is an increasing function; using an argument identical to Holmstrom’s (1979) and Shavell’s (1979) we can show that $\mu$ is strictly positive; $\nu$ is non-negative by the Kuhn-Tucker theorem; $f(Q^*; I) > 0$ since it is a density function. Hence the Monotone Likelihood Property implies that

$$\left[\nu f(Q; I) + \mu f_i(Q; I)\right] * \left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] > 0 \quad \text{for } Q < Q^*$$

$$\left[\nu f(Q; I) + \mu f_i(Q; I)\right] * \left[G(W + \alpha(Q) - C) - G(W + \alpha(Q))\right] < 0 \quad \text{for } Q > Q^*$$

It follows that we need $k(Q) < 0$ for $Q < Q^*$ and $k(Q) > 0$ for $Q > Q^*$ in order for (13) to hold. Since $\gamma(Q) < 0$ and $\beta(Q) > 0$ by the Kuhn-Tucker theorem, we obtain that $\Phi(Q) = 1$ for $Q < Q^*$ and $\Phi(Q) = 0$ for $Q > Q^*$.

Q.E.D.

Proposition 1 allows us to make a number of interesting conclusions about the optimal firing rule. First, firing is indeed helpful in controlling the agents as long as there exists a
positive cost to them of being fired. If there is no cost of this kind, firing does not improve
the principal’s position because the payment schedule α(Q) itself includes the possibility
of punishing the agent for low output with α = 0; firing with zero cost is implicitly built
into such a payment regime. Second, in cases when a positive cost of being fired exists
for the agent, we find that a deterministic rather than probabilistic firing rule is optimal:
the principal should set a target level of production (a task) Q∗, and then fire the agent
with certainty if he fails to accomplish the task and not fire him if he succeeds. From
now on, we will refer to Q∗ as to the optimal firing threshold, the target level or the task
set by the principal.

The next question we may ask is about location of the optimal firing threshold. Corollaries 1 and 2 below consider the location of Q∗ relative to the “peak” of the
distribution of output for the cases when the voluntary participation constraint is and is
not binding.

**Corollary 1** The optimal firing threshold Q∗ is never higher than the point ̂Q at which
f I(Q; I) changes sign: Q∗ ≤ ̂Q.

**Proof** From equation (11) defining Q∗ it follows that f I(Q∗; I) ≤ 0 since μ > 0, ν ≥ 0
and f(Q∗; I) > 0 as it was stated above. Hence by Lemma 1 we have Q∗ ≤ ̂Q, the optimal
firing threshold is not higher than the “peak” of the distribution of possible outcomes.

Q.E.D.

**Corollary 2** If the voluntary participation constraint is not binding and there is a posi-
tive cost to the agent of being fired, then Q∗ = ̂Q: the optimal firing threshold is located
at the “peak” of the distribution of the outcomes.

**Proof** If the voluntary participation constraint is not binding, i.e. ν = 0, condition (11)
implies:

\[ -μ * f I(Q∗; I) = 0. \]

Q.E.D.

The above corollaries help to explain the role of the voluntary participation constraint
in the choice of the firing threshold. If the constraint is not binding the principal will set
the firing threshold as high as the “peak” of the distribution of the agent’s output (given
that the agent chooses the investment level to maximize his expected utility), Q∗ = ̂Q(I).
The presence of the voluntary participation constraint induces the principal to make the

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firing rule less demanding, and may even lead her to choose the target level very close to zero, which would be practically equivalent to not using firing at all.

We now turn to other elements of the optimal contract. To analyze the properties of the optimal payment schedule, we consider the role of the limited liability constraint and whether it is ever binding: When does the agent get a zero share of the output?

**Proposition 2** Under the condition that the Monotone Likelihood Property holds, i.e., \( f_t(Q; I)/f(Q; I) \) is increasing in \( Q \), there exists a value \( \hat{Q}(I) \), \( \hat{Q}(I) > Q^*(I) \), where \( Q^*(I) \) is the optimal firing threshold, such that \( \alpha(Q) = 0 \) for any \( Q \leq \hat{Q}(I) \), and \( \alpha(Q) > 0 \) and increases with \( Q \) for \( Q > \hat{Q}(I) \). \( \hat{Q} \) is the solution to the equation:

\[
-V'(Q) + \mu G'(W) \frac{f_t(Q; I)}{f(Q; I)} = 0. \tag{14}
\]

**Proof** Consider condition (8) for the optimal \( \alpha(Q) \). Using the results concerning \( \Phi(Q) \) obtained in Proposition 1, we can rewrite (8) as the two following conditions:

\[-V'(Q - \alpha(Q)) + G'(W + \alpha(Q) - C) \* [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] + \eta(Q) = 0 \text{ for } Q < Q^*, \]

\[-V'(Q - \alpha(Q)) + G'(W + \alpha(Q)) \* [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] + \eta(Q) = 0 \text{ for } Q \geq Q^*. \]

As stated above, the value of the Lagrange multiplier \( \mu \) is strictly positive and \( \nu \) is non-negative; \( \eta(Q) \) is as well non-negative by the Kuhn-Tucker theorem. Observe that \( \eta(Q) > 0 \) and, consequently, \( \alpha(Q) = 0 \) when

\[-V'(Q - \alpha(Q)) + G'(W + \alpha(Q) - C) \* [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] < 0 \text{ for } Q < Q^*, \]

\[-V'(Q - \alpha(Q)) + G'(W + \alpha(Q)) \* [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] < 0 \text{ for } Q \geq Q^*, \]

or, substituting for \( \alpha(Q) = 0 \),

\[-V'(Q) + G'(W - C) [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] < 0 \text{ for } Q < Q^*, \tag{15}\]

\[-V'(Q) + G'(W) [\nu + \mu \frac{f_t(Q; I)}{f(Q; I)}] < 0 \text{ for } Q \geq Q^*. \tag{16}\]

As in the standard moral hazard problem without a limited liability constraint or firing rule, if the Monotone Likelihood property holds, i.e. if \( f_t(Q; I)/f(Q; I) \) is increasing in
Q, then, within each interval $Q < Q^*$ and $Q \geq Q^*$, both $\alpha(Q)$, when the limited liability constraint is not binding, and the left-hand sides of (15) and (16) are increasing in $Q$. Hence we can find values $Q^*$ and $\tilde{Q}$ at which the left-hand sides of (15) and (16), respectively, change signs and become non-negative. Now there are two possible cases:

1. $Q^* < Q^*$, in which case we could expect a discontinuity of $\alpha(Q)$ at $Q = Q^*$, since $\alpha(Q)$ would be positive for $Q^* < Q < Q^*$;

2. $Q^* \geq Q^*$, which would show that $\alpha(Q) = 0$ for all $Q \leq Q^*$ and for all $Q^* \leq Q \leq \tilde{Q}$, which would imply in turn that $\alpha(Q) \geq 0$ is continuous and non-decreasing over the whole range of $Q$.

We now show that 2 is indeed the case. Consider inequality (15). Since $V'(\cdot) > 0$ and $G'(\cdot) > 0$ by assumptions, the left-hand side of (15) can be non-negative only if

$$\nu + \mu \frac{f_I(Q; I)}{f(Q; I)} > 0,$$

or

$$\frac{f_I(Q; I)}{f(Q; I)} > -\frac{\nu}{\mu}. \quad (17)$$

By Proposition 1, the optimal firing threshold $Q^*$ is the solution to the equation:

$$\frac{f_I(Q^*; I)}{f(Q^*; I)} = -\frac{\nu}{\mu}. \quad (18)$$

Expressions (17) and (18) together with the Monotone Likelihood Property imply $Q^* > Q^*$ which shows that case 1 is impossible. Hence we are left with case 2: $\alpha(Q) = 0$ for any $Q \leq \tilde{Q}$, with $\tilde{Q} \geq Q^*$. Starting at $Q = \tilde{Q}$, the limited liability constraint is not binding any more, and $\alpha(Q)$ increases in $Q$.

Q.E.D.

Corollary 3 If the voluntary participation constraint is not binding, then $\tilde{Q} > \hat{Q}$, where $\hat{Q}$ is the point where the distribution has its "peak": $f_I(\hat{Q}; I) = 0$.

Proof If the voluntary participation constraint is not binding, that is, $\nu = 0$, the first order condition (8) implies that $\eta(Q) > 0$, i.e. $\alpha(Q) = 0$, as long as:

$$- V'(Q) + \mu G'(0) \frac{f_I(Q; I)}{f(Q; I)} < 0. \quad (19)$$
Now, by Lemma 1 for $Q < \hat{Q}$ we have $f_1(Q; I) < 0$, which implies that the left-hand side of (19) is negative. Hence, $\alpha(Q) = 0$ for $Q \leq \hat{Q}$. As in Proposition 2, we can show that the left-hand side of (19) is increasing in $Q$, hence there exists some $\hat{Q} > \hat{Q}$ at which the left-hand side of (19) becomes non-negative. From this point, the payment $\alpha(Q)$ starts increasing from zero.

Q.E.D.

**Corollary 4** Whether the voluntary participation constraint is binding or not, if the Monotone Likelihood Property holds, the optimal firing threshold is always lower than the levels of output at which the agent is paid a positive share of profit: $Q^* < \hat{Q}$.

The above observations concerning the optimal payment schedule $\alpha(Q)$ help one to understand the role of the limited liability and connect the elements of the optimal contract together. Proposition 2 shows that the function of the limited liability constraint is to keep the principal from imposing fines on the agent for low outcomes as happens in optimal risk sharing arrangements without limited liability. However, for higher outcomes, when the limited liability constraint is not binding, the profit sharing arrangement has the same feature as in a standard moral hazard problem: the agent is paid more for higher outcomes.

Corollary 3 shows the role of the voluntary participation constraint: when the constraint is present, the principal has to start paying positive amounts of money to the agent for lower outcomes. In the absence of this constraint, the agent has to produce an outcome above the mode of the distribution to be paid a positive share. Obviously, the more binding is the voluntary participation constraint, the wider is the region of outcomes for which the agent is paid.

Corollary 4 connects the elements of the optimal contract together, indicating how the optimal firing rule is chosen relative to the level of output for which the principal starts paying the agent with positive shares of output. We conclude that it is never optimal to reward the fired agent.

Given the above results we are in position to consider the sufficient conditions for an interior solution of the problem (1)-(6) which guarantee that the agent never steals or invests all the resources.

**Corollary 5** The agent’s choice of investment is always in the interior of the domain, $0 < I < R$, as long as $W \leq C$. 

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**Proof** Condition (6) implies the following possible solutions for the optimal $I$ (we denote it by $\hat{I}$):

(i) $\hat{I} = 0$ if $U''(0) \leq 0$

(ii) $0 < \hat{I} < R$ if \[
\begin{aligned}
&U''(0) > 0 \\
&U''(I) = 0 \text{ for some } I, 0 < I < R \\
&U''(R) < 0
\end{aligned}
\]

(iii) $\hat{I} = R$ if $U''(R) \geq 0$

In the context of our model and in view of the results obtained in Proposition 1, the expressions (i) and (iii) are:

(i) $\hat{I} = 0$ if $-g'(R) + \int_{0}^{Q'} G(W + \alpha(Q) - C)f_{I}(Q;0)dQ + \\
\quad + \int_{Q'}^{\infty} G(W + \alpha(Q))f_{I}(Q;0)dQ \leq 0$

(iii) $\hat{I} = R$ if $-g'(0) + \int_{0}^{Q'} G(W + \alpha(Q) - C)f_{I}(Q;R)dQ + \\
\quad + \int_{Q'}^{\infty} G(W + \alpha(Q))f_{I}(Q;R)dQ \geq 0$

However, case (iii) is impossible since by earlier assumptions $g'(0) = \infty$, and $-\infty < f_{I}(Q;I) < \infty$ for any $Q$, $I$. Now consider case (i). Note that by assumptions $-g'(R) = 0$ and $\hat{Q}(I = 0) = 0$, hence $Q^{*}(I = 0) \geq \hat{Q}(I = 0)$ and $f_{I}(Q;0) \geq 0$ for any $Q \geq Q^{*}$, which together with $G(x) > 0$ for $x > 0$ shows that the sum of the first and the last terms in the left-hand side of the expression in (i) is always positive. Thus the case (i) is impossible if the second term is also non-negative. Consider this term. By Corollary 4 $\alpha(Q) = 0$ for $Q \leq Q^{*}$; hence if $W \leq C$ we get

$$
\int_{0}^{Q'} G(W + \alpha(Q) - C)f_{I}(Q;0)dQ = \int_{0}^{Q'} G(W - C)f_{I}(Q;0)dQ = \\
= G(W - C)\int_{Q}^{Q'} f_{I}(Q;0)dQ = G(W - C)F_{I}(Q^{*};0) \geq 0
$$

since by assumption $F_{I}(Q;I) < 0$ and $G(x) \leq 0$ for $x \leq 0$. Thus under the stated condition the only possibility is (ii), the interior solution for $I$, with $U''(I) = 0$.

Q.E.D.

We can conclude that a high level of fixed payment to the agent creates more incentives for him to shirk. Thus if the principal for some reason has to pay a lot to the agent
no matter how he performs (the reason can be, for example, an ex-post voluntary participation constraint), she, in some cases, might not even be able to induce the agent to invest any resources into task implementation. And, visa versa, a low level of guaranteed payment gives the principal more control over the agent.

Finally, we give a summary of the properties of optimal firing contracts with and without a voluntary participation constraint. As Propositions 1 and 2 show, the optimal firing strategy for the principal is to set a certain task for the agent and fire him if and only if he fails to accomplish the task. The task, or the firing threshold, is never set above the “peak” of the distribution of the outcome. If there is a voluntary participation constraint, the firing rule becomes more tolerant; however, it is never possible that the agent gets fired and is still paid a positive share of the output. If the voluntary participation constraint is not binding, then the optimal firing threshold is set at the “peak” of the distribution of the agent’s output, and the agent still has to produce higher output to get paid a positive share.

In the next section, we consider some comparative statics results for the agent’s problem.

4 Comparative statics for the agent

We are interested in discovering how sensitive our results are to the exogenous variables of the model, such as the cost of being fired $C$ and the total amount of resources $R$ available to the agent. For the first step, we consider how these variables affect the agent’s choice of investment $I$, with other parameters of the contract being fixed. For this purpose, we go back to the solution of the agent’s problem. The first order conditions are:

\[ U'(I) = -g'(R-I) + \int_0^{Q^*} G(W+\alpha(Q)-C)f_I(Q;I)dQ + \int_{Q^*}^{\infty} G(W+\alpha(Q))f_I(Q;I)dQ = 0 \]  

(20)

As in a standard moral hazard problem without firing, convexity of the distribution function $F(Q;I)$ and the Monotone Likelihood Property guarantee that the first order conditions (20) are also sufficient for the maximum\footnote{For the proof, see for example Mirrlees’ and Rogerson’s approach as presented in Laffont (1989).}. Thus we can turn directly to the question of how the agent’s choice of the investment level changes with the cost of being fired and the total amount of resources, other things being equal.

**Proposition 3** Other things being equal, the agent’s optimal choice of investment $\tilde{I}$ is an increasing function of both the cost of being fired $C$ and the total amount of available resources $R$. 

Proof Using the standard comparative statics technique, we differentiate the equality (19) with respect to $C$ and $R$, which implies:

Given $U_{II} < 0$, $I_C > 0$ if and only if $U_{IC} > 0$ ;
Given $U_{II} < 0$, $I_R > 0$ if and only if $U_{IR} > 0$ .

Assume the distribution function $F(Q; I)$ is convex and therefore $U_{II} < 0$. Then, using the properties that the distribution function is changing in $I$ in the sense of First Order Stochastic Dominance, i.e. $F_I(Q; I) < 0$, and $g(R - I)$ and $G(\cdot)$ are increasing and concave, we obtain

\[
U_{IC} = -\int_0^{Q^*} G'(W + \alpha(Q) - C) f_I(Q; I) dQ \geq -\int_0^{Q^*} G'(W + Q^*) f_I(Q; I) dQ \\
\geq -G'(W + Q^*) * F_I(Q^*; I) > 0, \tag{21}
\]

\[
U_{IR} = -g''(R - I) > 0. \tag{22}
\]

It follows that both $I_C > 0$ and $I_R > 0$.

Q.E.D.

Proposition 3 is intuitive: with the cost of being fired rising, the only way that the agent can try to avoid this cost is by investing more in the principal’s task, and thus decreasing the probability of being fired. Also, with the total amount of available resources increasing, we can expect the agent to increase both the investment and the amount he steals to keep the first order conditions in balance.

The next problem we consider is how the agent’s choice of investment depends on the firing rule set by the principal, other things being fixed. More specifically, we are interested in explaining the principal’s optimal choice of the firing threshold $Q^*$.

Proposition 4 Other things being equal, the agent’s choice of the investment level $\hat{I}$ is increasing in $Q^*$ when $Q^* < \bar{Q}$, not changing around $Q^* = \bar{Q}$, and decreasing for higher values of $Q^*$, where $\bar{Q}$ is the point such that $f_I(\bar{Q}; I) = 0$.

Proof Using the same comparative statics technique as in Proposition 3, we get

\[
I_{Q^*} = -U_{IQ^*}/U_{II}
\]

Given $U_{II} < 0$, $I_{Q^*} > 0$ if and only if $U_{IQ^*} > 0$

\[
U_{IQ^*} = [G(W + \alpha(Q^*) - C) - G(W + \alpha(Q^*))] * f_I(Q^*; I) \tag{23}
\]
Since $G(\cdot)$ is an increasing function, the sign of $U_{IQ^*}$ and consequently the sign of $I_{Q^*}$ depend only on the sign of $f_i(Q^*; I)$. Hence by Lemma 1:

\[
\begin{cases}
I_{Q^*} > 0 & \text{for } Q^* < \hat{Q} \\
I_{Q^*} = 0 & \text{for } Q^* = \hat{Q} \\
I_{Q^*} < 0 & \text{for } Q^* > \hat{Q}
\end{cases}
\] (24)

Q.E.D.

Proposition 4 shows that $I$ achieves its maximum when $Q^* = \hat{Q}$, other things being equal. This result helps to explain the choice of $Q^* = \hat{Q}$ as an optimal firing threshold in the absence of the voluntary participation constraint. Setting this firing rule, the principal gains because $Q^* = \hat{Q}$ induces the highest investment on the part of the agent. Besides, by Proposition 3, the presence of a positive cost of being fired induces a higher investment level than would occur in the absence of this cost. However, if the voluntary participation constraint is binding, the principal has to give up her preferred firing rule to induce the agent to participate in the contract. Still the firing rule helps the principal to control the agent. To summarize, it must be the case that the increase in investment induced by the firing rule does not require a correspondingly higher increase in expected payment. Then the principal will be better off compared to the situation without firing.

\section{Conclusion}

The preceding analysis shows that, in the presence of limited liability, when the profit sharing contract is combined with the firing rule it improves the principal’s position if there is a cost of being fired for the agent. However, if there is no cost, the firing rule is not helpful since it does not provide the principal with a new instrument of regulation. In the case of a positive cost, the agent invests more resources in production if there is a firing threat. The optimal policy for the principal with respect to the firing rule is not to randomize the firing decision for every possible outcome but to set a certain target level of output and then fire the agent with certainty in case he performs below this level. This conclusion is consistent with Zou’s (1992) result that threat-incentives contracts, where the agent receives different payments depending on whether he has performed above or below a certain target level, approximate the optimal solution to the moral hazard problem.

The arrangement we have considered can be viewed as type of contract with a warranty where the agent has to pay a fine (a warranty) in case the principal is not satisfied with the agent’s product. The obvious difference is that in our case the “warranty” is not
a transfer from the agent to the principal, but an exogenous cost paid by the agent. The principal cannot directly profit from this cost, nor can she choose its magnitude; yet her power to impose this cost on the agent appears to be effective in disciplining the agent.

The principal should set the firing rule not higher than the certain level of the output which in many cases corresponds to the peak of the distribution of random outcomes. In this case, the agent is induced to invest more than in the contract without firing, which makes the principal better off. Besides, the principal may gain by having to pay the agent less — at least in expectation, — than he would have to do otherwise.

The higher the agent's cost of being fired, the stronger his incentives to make larger investments. Thus, the effectiveness of the firing rule as a controlling device also depends on the cost of being fired.

If the voluntary participation constraint is not binding, the principal has more power in controlling the agent. In this case, it is optimal to set the firing rule at the “peak” of the distribution of the possible outcome and to start paying positive shares to the agent for higher levels of output than the firing rule.

We find that it is never optimal for the principal to reward the fired agent with a positive share of output. This conclusion is intuitive and is also consistent with Chwe (1990) who found that the principal should not use rewards and punishments at the same time. In his setup, however, this result followed from it being costly to the principal to impose both rewards and punishments. As we find in this paper, the result still holds in many situations even though punishment is costless to the principal.

We can now come back to the examples presented in the introduction. In application to managerial control in transition economies the preceding analysis implies that firing may not be enough to prevent stealing. If the current manager does not have strong career concerns, it may be hard to deter him from stealing the firm's resources. In this case more careful monitoring might be required; besides, it may be possible to fire the manager at the beginning of the transition process, if his behavior is detected early. On the other hand, if the cost of being fired is substantial, the introduction of a firing rule may indeed induce better performance and thus help to accomplish the transition measures.

We should note however that the proposed arrangement may be helpful only under specific circumstances when the contracts proposed to the agent can be made binding and the firing threat - credible. Unfortunately this is not likely to be the case for transition economies: in the situation with an excessively high level of economic uncertainty and poorly defined property rights enforcibility of contracts becomes a very serious problem. On the other hand, reputation concerns may be still essential in the case of "one-shot deals": new economic agents usually care about their business image.
With respect to the procurement and “pre-retirement” examples, the principal’s ability to impose a reputation cost on the agent certainly appears to be a way to reduce the “last period effect” of the relationship. If a contractor or an employee near retirement cares about the reputation that they get by the end of the contract, which is often the case, the principal should be aware of this fact and use it to her advantage. Also, enforcibility of contracts is quite a manageable issue in these situations.

There are a number of possible extensions of the analysis presented in this paper. First, it might be interesting to consider a multistage model, when the agent can be fired after every stage, and compare it to existing repeated principal-agent models (see, for example, Banks and Sundaram (1991)). Also, it is possible to consider optimal contracts for similar circumstances when agents can differ in types. We have covered the latter question slightly in this paper by discussing the role of the cost of being fired: different levels of costs may represent different types of agents. Finally, it is interesting to consider a situation when the principal can choose the level of resources that she gives to the agent or, alternatively, when the supply of resources to the agent is itself random and unobservable by the principal. The latter two extensions of the problem are of special interest.
References


