SUBCOMPONENT INNOVATION AND MORAL HAZARD: WHERE TECHNOLOGICAL PROGRESS MEETS THE DIVISION OF LABOR

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ABSTRACT

I model the technical innovation of a final good as a process of incremental enhancement due to Research and Development (R&D) efforts undertaken on subcomponents to the final good. R&D contracting is analyzed within various principal/agent structures. I identify a principal who jointly values the performance capabilities of the subcomponent undergoing R&D and the funds available for other subcomponents; thus, he does not have a transferable utility function. I justify and characterize a performance seeking agent in addition to the conventional profit seeking agent. The information environment and the motivational properties of the principal and agent significantly affect the form and existence of optimal R&D contracts. I draw insights for private and public sector industrial organization.
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1.0 INTRODUCTION

The fundamental effect of technological progress on production is obvious and well accepted. It is also clear that, in a world of complex final goods, technological progress proceeds at the subcomponent level, with innovations incrementally incorporated into final goods. Innovation is fraught with greater uncertainties than established production; thus, the agency concerns that influence industrial organization and optimal contracting are surely relevant. The optimal production organization for the technological evolution of a final good must therefore be fundamentally affected by the relationship between subcomponent innovator and final good producer. However, contractual arrangements for subcomponent innovation, hereafter referred to as Research and Development (R&D), are not adequately addressed by either the existing Industrial Organization or Optimal Contracting literatures, leaving this critical intersection between the division of labor and technological progress under-addressed.¹

By expressly considering the subcomponent nature of production while emphasizing the goals of the final good producer and the goals of subcomponent innovators I obtain a useful and significant characterization of optimal R&D production organization. I begin by classifying the contractual forms observed for subcomponent R&D procurements, then I characterize the

¹Coase's observation (1937) that production is vertically integrated to the degree that the marginal cost of so doing is below that of market contracting provides a unifying paradigm for production organization yet so borders on tautology that little can be specifically explained. Alchian's and Demsetz' corollary (1972) that production is vertically integrated until the cost of management forces the marginal cost of internal production to exceed that of market procurement adds little except to suggest that efficiency within a firm is degraded when subordinates can conceal inputs to production, thus extracting a rent from management. Williamson (1979) expands on Alchian's, Crawford's, and Klein's observation (1978) of the significance of bilateral monopoly in complex subcomponent production by illustrating the effects of moral hazard in both internally organized and market contracted production. Past analytical research has concentrated on principal/agent models of single component production with no work expressly considering the effects of subcomponent production.
subcomponent nature of the final product, the interests of various principals, and the interests of the potential agents. I proceed through a structured, analytical modeling of the types of R&D production organization observed to identify general optimal contracting arrangements. A primary goal of this research is to provide practical insight into how R&D should be organized; therefore, I conclude the paper with a procurement policy section.

2.0 OBSERVED APPROACHES TO R&D PROCUREMENT

The pre-R&D existence of a final good composed of numerous subcomponents implies that a decision to innovate one subcomponent will result in its post-R&D incorporation only if the outcome of the R&D results in superior performance to the existing subcomponent. Further, to warrant an R&D investment, a range of performance outcomes weighted significantly above the status quo must be expected. Therefore, it is reasonable that R&D subcomponent procurement contracts may be variable in both price and performance. The acceptability of a range of performance outcomes is an important characteristic distinguishing subcomponent R&D from standard procurement. Within this variable performance structure, the following three general contract forms encompass the organizational approaches observed between the final good producing principal and the subcomponent R&D agent.\(^2\)

**Pure Fixed Price [PFP] --** The agent receives a set payment regardless of the R&D performance outcome

**Fixed Price Performance Based [FPPB] --** Based on the performance outcome of R&D, the agent receives a payment agreed upon prior to his commencing R&D.

**Cost Exposure Sharing [CES] --** Based on interim progress, the agent receives payment relative to a price/performance relationship specified prior to his commencing R&D. This provides the principal with information to more efficiently configure the other subcomponents to the final good while providing the agent with cost risk insurance.

The timing of payments under any of these contract forms will vary with the agency structure and the public or private sector affiliation of the principal: e.g., progress payments from the principal will be required for both internal agency and U.S. federal government procurement;\(^3\) whereas, payment timing and contingent rebates (warranties) may be negotiated between a private principal and a private external agent. However, given \textit{ex ante} complete

\(^2\)All contracts considered in this research are \textit{ex ante} complete. A discussion of the relevance of incomplete contracts that require interim or \textit{ex post} renegotiation is included in Appendix A. More complete and applied descriptions of R&D contracts observed in practice are included in section 5 where the results of the following analysis will be brought to bear on observed industrial organization.

\(^3\)Due to the Anti-Deficiency Act, a federal agency acting as the principal in a procurement must provide progress payments sufficient to allow for contract termination at the government's discretion without subsequent financial liability to the federal government.
contracts, payment timing is an accounting issue rather than a substantive one. The substantive issues for R&D subcomponent procurement are agency concerns, particularly moral hazard. Through analyses of principal/agent structures varying in information makeup and participant motivations I characterize optimality among these three general contract forms.

3.0 R&D PROCUREMENT FROM A PROFIT SEEKING AGENT

My analysis models R&D as a two step process, influenced by a correlation between the probabilistic nature of innovation and the interests of the involved parties. In section 3, I examine R&D production within the context of conventional profit seeking agency: I introduce the basic R&D production process removed from agency complications (subsection 3.1). To ground my analysis in the existing literature, I characterize optimal R&D contracting between a principal contracting for a complete good from a profit seeking agent (subsection 3.3). I make my first significant adaptation to conventional principal/agent analysis by attempting to characterize an optimal contract between these same parties but regarding a subcomponent to final production rather than the complete good (subsection 3.4). Profit seeking agency is then rounded out by introducing the wholly reasonable prospect of interim adverse selection (subsection 3.5). Doubt is cast on the existence of optimal R&D contracting arrangements that allow the principal to benefit from mid-production information about the outcome of the agent’s Research effort. This doubt motivates the analysis of section 4 involving an agent who is at least partially motivated by the performance outcome of the subcomponent that he is contracted to innovate.

3.1 R&D Production without Agency Concerns [analysis of the first best]

Research involves an effort to enhance the technological inputs to a subcomponent while Development incorporates the technological outcome of Research into actual subcomponent production. R&D has been modeled as an iterative process, where Research efforts are expended until an acceptable prototype of a final good results, which is then Developed. Alternatively, a single Research endeavor may be undertaken with the intent of innovating a subcomponent to a many component final good. By this alternate model, the innovation will be Developed into the necessary subcomponent if it will result in value superior to available Off-the-Shelf substitutes. The latter process does not preclude further R&D on the same subcomponent or any other, it simply reflects the incremental technological evolution of a complex production process. As we wish to model subcomponent production, we adopt the more incremental definition of R&D.

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Figure 3.1 provides a four period model of R&D production without agency concerns. Each period represents an outcome, with the paths between periods representing decisions that affect the next outcome. Under the assumption that the principal is cognizant of the nature of the whole process it is appropriate to describe Figure 3.1 from the period 3 outcome backward:

The period 3 outcome is a subcomponent with performance capability $S$, where $S$ depends on the outcome of Research, $\tau$, and Development funding, $t$. Production of the final good proceeds after period 3, where $S$ is combined with other subcomponents purchased using residual funds, $\Pi = B - \lambda - t$ [B arbitrary]. The principal values production of $S$ relative to all of the subcomponents comprising the final good; therefore, the principal's utility function is modeled as $U[S(\tau, t), \Pi]$.

The period 2 outcome is the technology, $\tau$, developed from the Research process. $\tau$ depends positively on the principal's Research investment decision, $\lambda$, and a probabilistic element, $\theta$, drawn from a publicly known distribution $F(\theta)$ with density $f(\theta)$ [i.e., $\tau = \pi(\lambda, \theta)$]. The minimum $\tau$ is fixed by the pre-Research, Off-the-Shelf, level of technology and denoted as $\tau_0$. Upon observing $\tau$, the principal decides what level of Development funding, $t(\tau)$, to invest in the production of $S$.

The period 1 outcome is that a Research effort funded by an investment $\lambda$ is underway. Nature makes the probabilistic decision $\theta$ that will affect the Research process.

The period 0 outcome is that the principal has concluded that the subcomponent, $S$, should undergo R&D. The principal decides on a Research investment of $\lambda > 0$.$^5$

**Fundamental Management Postulate:** The decision to modify final good production by innovating subcomponent $S$ is made with the expectation of increased value to the principal.

**Implications:** In the absence of agency concerns, the R&D decision is governed by the available technological possibilities for all subcomponents, the principal's budget, and the principal's valuations. In the presence of agency concerns, innovating $S$ becomes a management decision over a technical process that will be manipulated by an agent; implying that innovation of $S$ is *ex ante* warranted given compensatory rents for moral hazard and adverse selection. All of the following analyses thus concentrate on whether or not the solutions proposed by first order optimization techniques satisfy the technological and agency requirements for $\lambda > 0$; assuming that, in expectation, the principal is better off for having made the management decision to commence R&D of $S$.

$^5$The following functional characteristics specify the nature of this R&D process:
- Outcome of Research is technology $= \pi(\lambda, \theta)$
  - $\tau(\lambda, 0)$ is concave $\tau_1 > 0$, $\tau_0 < 0$, $\tau(0, 0) = \tau_0$ [Off-the-Shelf] for all $\theta$, $\tau(\lambda, 0) \geq \tau_0$ for all $\lambda \geq 0$ [revert to $\tau_0$ if 0 outcome is 'bad'], $\tau(\lambda, 0) \leq \tau_{\text{max}}$ for all $\lambda > 0$ [support of $f(\theta)$ is independent of $\lambda$]
- Outcome of Development is the subcomponent $S(\tau, t(\tau))$
  - $S$ is concave $S_0 > 0$, $S_{\text{Ii}} < 0$, and $S_{\text{II}} > 0$, $t(\tau)$ is Development funding, $S(\tau, 0) = 0$ for all $\tau$.
  - Principal's Utility $= U(S, II)$. $U$ is concave in both arguments $U_1 > 0$, $U_\text{II} < 0$.
  - $S$ represents a subcomponent undergoing R&D, II is funding for Off-the-Shelf sub-components
- The budget for production of the final good is $B$ [arbitrary for our purposes]
Solving backwards -- Second Period Production Funding, \( t(\tau, \lambda, \theta) \):

\[ \mathcal{L}_2 = U[S(\tau, t), B - \lambda - t] + \gamma t \quad \gamma = 0 \text{ if } t > 0, \gamma = 0 \text{ if } t = 0 \]

\[ \frac{d\mathcal{L}_2}{dt} = S\lambda U_S - U_x + \gamma = 0 \quad \forall (\lambda^*, \theta) \]

The solution \( t^* \) to (3.2) will be a maximum if

\[ \frac{d^2\mathcal{L}_2}{dt^2} = S\mu U_S + S^2 U_{SS} - 2S\lambda U_{S\theta} + U_{\theta\theta} \leq 0 \]

Note that given the concavity of \( U \) and \( S \), (3.3) is less than zero if \( U_{S\theta} \geq 0 \). \( U_{S\theta} \geq 0 \) is a mathematical representation of the positively and jointly valued nature of the various subcomponents in final good production; namely, the marginal value of increasing the capability of one subcomponent is increasing in the capability of all other subcomponents. Henceforth, this property will be referred to as balance, in the sense that the principal wishes to balance the constituent worth of the subcomponents as inputs to the final good.

Assumption 3.1: \( U_{S\theta} \geq 0 \) [the principal values balance].

Some comparative statics for the Development funding decision, \( t^* \), illuminate the R&D process and the effect of the Research outcome, \( \tau(\lambda, \theta) \), on the optimal funding decision:

\[ \frac{dt^*}{d\theta} = \frac{-\tau\theta(S\mu U_S + S\lambda U_{SS} - S\lambda U_{S\theta}) - \gamma \theta(\lambda, \theta)}{S\mu U_S + S^2 U_{SS} - 2S\lambda U_{S\theta} + U_{\theta\theta}} = \frac{d\pi^*}{d\theta} \]

\[ \frac{dt^*}{d\lambda} = \frac{-\tau\lambda(S\mu U_S + S\lambda U_{SS} - S\lambda U_{S\theta}) + S\lambda U_{S\theta} - U_{\theta\theta} - \gamma \lambda(\lambda, \theta)}{S\mu U_S + S^2 U_{SS} - 2S\lambda U_{S\theta} + U_{\theta\theta}}. \]

This implies that for \( (\lambda^*, \theta) \ni -S\mu S\lambda U_{SS} + S\lambda U_{S\theta} > S\mu U_S, \frac{dt^*}{d\lambda} \leq \frac{dt^*}{d\theta} \leq 0, \]

while for \( (\lambda^*, \theta) \ni -S\mu S\lambda U_{SS} + S\lambda U_{S\theta} < S\mu U_S, \frac{dt^*}{d\theta} \geq 0 \) and \( \frac{dt^*}{d\theta} \geq \frac{dt^*}{d\lambda} \).

Note that for any level of Research funding, \( \lambda \), Development funding, \( t \), may be decreasing in relation to the outcome of the Research, \( \tau(\lambda, \theta) \). Figure 3.2 illustrates the principal's utility space relative to Development funding. A potential \( t'(\tau) \) funding schedule is shown that is increasing and decreasing in \( \tau \). Development funding that decreases with the positive outcome of a learning phase is intuitively reasonable for a subcomponent to a final good, the value of which depends on the performance contributions of multiple subcomponents. This is implicit in the assumption that \( U \) is concave and has a non-negative cross product between \( S \) and \( \Pi \).
Next solve for the Research funding decision, $\lambda$, at Period 0:

$$
\mathcal{E}_0 = \int U[S(\tau(\lambda, \theta), t(\lambda, \theta)), B - \lambda - t(\lambda, \theta)]f(\theta)d\theta
$$

$$
\frac{d\mathcal{E}_0}{d\lambda} : \int [U_S[\tau_\lambda S_t + t_\lambda S_t] - U_\pi]f(\theta)d\theta = \int [\tau_\lambda S_t U_S - U_\pi + t_\lambda S_t U_S - U_\pi]f(\theta)d\theta \leq 0
$$

From (3.2): $S_t U_S - U_\pi = y^*(\lambda, \theta) \forall \theta$

$$
\frac{d\mathcal{E}_0}{d\lambda} : \int [\tau_\lambda S_t U_S - U_\pi - y^*(\lambda, \theta)t_\lambda^*]f(\theta)d\theta \leq 0
$$

From the Kuhn-Tucker conditions: $y^*(\lambda, \theta)t_\lambda^* = 0$; therefore

$$
\frac{d\mathcal{E}_0}{d\lambda} : \int [\tau_\lambda S_t U_S - U_\pi]f(\theta)d\theta \leq 0.
$$

From (3.11) we determine the second derivative of (3.8) with respect to $\lambda$

$$
\frac{d^2\mathcal{E}_0}{d\lambda^2} = \int [\tau_{\lambda \lambda} S_t U_S + \tau_{\lambda \theta} S_t U_S + (\tau_{\lambda \theta} S_t)^2 U_S - \tau_{\lambda \lambda} S_t U_S + \tau_{\lambda \theta} S_t U_\pi + U_\pi]f(\theta)d\theta \leq 0
$$

Theorem 3.1: The solution $(\lambda^*, t^*)$ to the R&D production model without agency concerns is the unique optimum.

Proof: Immediate from the second order conditions (3.3) & (3.12), and the linearity of the constraint in (3.1). QED
In the absence of agency concerns, R&D subcomponent production organization is a straightforward multi-period dynamic programming process with decisions contingent on an observable Research outcome. The implicit value from balancing the performance contributed from the R&D subcomponent with the performance obtainable from other purchased subcomponents results in the explicit fact that Development funding may be negatively related to the success of Research.

3.2 Profit Seeking Agency within the R&D Production Structure

Figure 3.3 illustrates the inclusion of a profit seeking agent in the R&D subcomponent production process. Research funding, \( \lambda \), is chosen by the agent relative to the common knowledge probabilistic quality of Research (i.e., the random variable \( \theta \) with density \( f(\theta) \)). The outcome of Research, \( \tau \), is assumed to be common knowledge; however, the inputs to Research, \( \lambda \) and \( \theta \), are observed by the agent only. Upon observing \( \tau \), the agent chooses Development funding, \( p(\tau) \). The production tradeoffs for \( S \) between \( \tau \) and \( p \) are assumed to be common knowledge; thus, the observation of \( \tau \) by the principal implies the observation of \( p \), and vice versa. Upon delivery of \( S \), the agent receives a payment based on a contract \( t(\tau) \). The principal values the qualities of the good \( S \) and funds, \( II \). The agent values simple profit; however, he may have limited access to agent-specific Research assets such that committing \( \lambda \)-worth of these assets represents an opportunity cost beyond transferable monetary expense. The function \( \psi(\lambda) \) is included in the agent’s utility function, \( V \), to account for this opportunity cost.

The R&D contract, \( t \), in Figure 3.3 may depend on an observation of the Research outcome, \( \tau \), in a manner affecting purchase price, performance capability and agency profit. Referring back to section 2, this role of an interim outcome is in accordance with a Cost Exposure Sharing (CES) contract. Given the \textit{ex ante} general contracting structure, the interim production report of \( \tau \) establishes the performance/price tradeoff point to be purchased by the principal and the division of Research-related financial risk borne by principal and agent. In this research, I will not determine the precise form of any optimal CES contract; rather, my analyses will be based on

![Diagram of R&D Production Process](image-url)
characterizing optimal incentive compatible direct revelation (ICDR) contracts. The Revelation Principle assures the existence of an optimal real-world analog to an optimal ICDR contract. By characterizing the optimal ICDR contracts for R&D production processes, I will draw insights into optimal real-world contracting.

A one-stage production process captured by periods 0, 1, 2, and 4 of Figure 3.3, and simplified with a profit motivated principal contracting for a complete and marketable good $S$ [e.g.; $U = U(S - S)]$, is essentially the simple moral hazard process studied by Ross [1973], Holmström [1979], and others. These authors have relied on the Revelation Principle to validate ICDR approaches that employ analysis of first order optimization conditions. Although the functional forms chosen and the questions of interest have varied, each of these authors have been able to characterize the optimal contract as positively monotonic in the outcome of the complete good that the agent was contracted to produce. Positive monotonicity is intuitively reasonable as both principal and agent can be unambiguously made better off by increasing output in models with transferable utility.

The two-stage, production process shown in Figure 3.3, but simplified with $U = U[S - t]$, is markedly similar to the R&D model of Guofu Tan [June 1989]. Tan models both single and multiple agent cases. His single agent case is the one of interest here. His model's fundamental difference from mine is that the outcome of Research, $r$, along with the inputs to research, $λ & θ$, are not observable, only the final output, $S$, is observable. Thus, the root moral hazard problem is augmented by an interim adverse selection problem. The two models diverge further in the functional restrictions that Tan imposes: agent and principal are risk neutral, $U$ is linearly separable in $S$ and $H$, and $V$ is linearly separable in $t$ and $λ$. All the same, the significance of information asymmetries in R&D procurement is highlighted in Tan's work and he is able to show results similar in spirit to the body of the more standard research. In this and the following subsection, I present a version of R&D procurement without the interim adverse selection problem, but with a principal and agent that may be risk averse. In subsection 3.5, I examine the interim adverse selection effect of an unobservable Research outcome.

3.3 R&D Procurement of a Final Good from a Profit Seeking Agent

To provide a baseline for the novelty of my work, it is appropriate that I employ my nomenclature and production phasing to characterize the type of optimal R&D contracting examined in the traditional literature; i.e., a contract with a profit seeking agent for a complete

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7Tan's production environment is similar, including an equivalent definition of Off-the-Shelf tech., $r_0$. 

8
good that is directly valued by the principal. Referring to Figure 3.3, \( p(\tau) \) is a monitorable development expenditure made by the agent which, when combined with the outcome of Research, \( \tau \), yields \( S(\tau, p) \). Gross compensation to the agent is \( t \) and net compensation is \( t - p \). If the principal is procuring \( S \) as a final, complete good, then the following specify the traditional R&D contracting problem in the context of the basic process introduced in subsection 3.1:

(i) Suppress \( \theta \) by viewing \( \tilde{\tau} \) as a random variable with distribution \( F(\tau, \lambda) \) and finite support \([\tau_0, \tilde{\tau}]\). Given a distribution of \( \theta \), \( F(\tau, \lambda) \) is the distribution induced on \( \tilde{\tau} \) via the relationship \( \tilde{\tau} = \pi(\lambda, \theta) \). Assume that \( F \) is convex in \( \lambda \); i.e., \( F_{\lambda} < 0, F_{\lambda \lambda} > 0 \).

(ii) \( S = S(\tau, p(\tau)), S_1 > 0, S_0 \leq 0, \text{ and } S_{\tau p} \geq 0 \)

(iii) \( V = v(t(\tau) - p(\tau)) - \psi(\lambda), U = U[S - t(\tau)], U', v' > 0, U'' \text{ & } v'' \leq 0, \psi'' \geq 0 \)

Agent’s Problem:

Maximize \( V \) w.r.t. Research cost: \[
\frac{d}{d\lambda} \left[ \int v'(t - p) - \psi(\lambda) \right] d\tilde{\tau} = 0 \Rightarrow \int v\lambda d\tilde{\tau} - \psi' = 0 \quad 3.13
\]

Principal’s Agency Constrained Problem:

\[
J = \int U[S - t(\tau)] d\tau + \gamma \int [v - \psi] d\tilde{\tau} + \mu \int v\lambda d\tilde{\tau} - \psi'' + \delta
\]

Participation Constraint \quad Incentive Compatibility Constraint

Necessary Condition for an optimal \( \lambda \)

\[
\frac{dJ}{d\lambda} = \int U[S - t(\tau)] f\lambda d\tau + \mu \int v\lambda d\tilde{\tau} - \psi'' = -\delta \quad 3.15
\]

Integrating by parts yields

\[
UF_{\lambda} b = \int_{a}^{b} F_{\lambda} U'[1 - t'] d\tau + \mu \left[ vF_{\lambda} b - \int_{a}^{b} F_{\lambda} t' v' d\tau + \psi'' \right] = -\delta \quad 3.16
\]

The nature of the distribution function, \( F(\theta, \lambda) \), allows for two useful simplifications:

\[
F(a, \lambda) = 0 \quad \forall \tau, F(b, \lambda) = 1 \quad \forall \tau; \quad \therefore UF_{\lambda} b = vF_{\lambda} b = 0 \quad \forall \tau
\]

Therefore, (3.16) can be rewritten as

\[
-\int F_{\lambda} U'[S_t + pS_p - t'] d\tau + \mu \left[ vF_{\lambda} \psi'(t' - p') d\tau + \psi'' \right] = -\delta \quad 3.17
\]

Necessary Condition for an optimal \( t(\tau) \)

\[
\frac{dt}{d\tau} = -\frac{U'}{v'} + \gamma v' + \mu v f' = 0 \quad \forall \tau \Rightarrow \frac{U'}{v'} = \gamma + \mu f' \quad \forall \tau \quad 3.18
\]

Necessary Condition for an optimal \( p(\tau) \)

9
\[
\frac{df}{dp} = \frac{S_p U'}{v'} = \gamma + \mu \frac{f}{I} \quad \forall \tau
\]

Comparing (3.18) with (3.19) yields

\[ S_p = 1 \quad \forall \tau \]

Differentiating each side of (3.20) with respect to \( \tau \) yields

\[ S_p + p'S_{pp} = 0 \quad \forall \tau \Rightarrow p' = -\frac{S_p}{S_{pp}} \geq 0 \quad \forall \tau \]

Differentiating each side of (3.18) w.r.t \( \tau \) and substituting for \( p' \) using (3.21) yields

\[ v'\left[U''(S_p + p'S_p - t') - v''(t' - p')U' = \mu v^2 \frac{df}{dt} \right] \]

Substituting in (3.20) and rearranging yields

\[ t' = p' + \frac{v'S_p U'' - \mu v^2 \frac{df}{dt}}{v'U'' + \mu v'} \quad \forall \tau \]

Assumption 3.2: \[ \frac{d}{dt} \left( \frac{f}{f} \right) \geq 0 \quad \text{[Monotone Likelihood Ratio Property, Standard]} \]

Given the concavity of \( U \) and \( v \), \( p' > 0 \) from (3.21), Assumption 3.2, and \( \mu > 0 \) [due to Holmström, 1979], equation (3.23) directly leads to the critical inequality for net compensation (\( t' - p' \)):

\[ (t' - p') \geq 0 \quad \forall \tau \]

Such a contract corresponds to a CES contract where the agent's profit is positively monotonic in his Research outcome, the agent is exposed to greater cost risk for less successful Research outcomes, and the final performance and price of the produced item are not specified until after the Research outcome is known.

From (3.17), a sufficient condition for the agent's maximization problem is that the transfer be positively monotonic; thus, the candidate optimal solutions solve the agent's problem. Similarly, from (3.17) and (3.20), a sufficient (yet overly strong) condition for an interior maximization to the principal's problem is:

\[ t' - p' \leq S_p \overset{(3.23)}{=} -\frac{v''}{v} \geq \mu v \left[ \frac{\frac{df}{dt}}{S_p U'} \right] \quad \forall \tau . \]

Condition (3.25) may be thought of as a mathematical representation of the Fundamental Management Postulate (FMP) for the complete good R&D case of Figure 3.3.

Theorem 3.2: The candidate ICDR solution \([\lambda^*, t^*(\tau), p^*(\tau)]\) to final good R&D production derived from the first order conditions [(3.15), (3.18), and (3.19)] is the unique optimum.
Proof: \( t' - p' \geq 0 \) implies that the constraints on (3.14) are convex in \( \lambda \). FMP (3.25) implies that the maximand of (3.14) is concave in \( \lambda \). The LeGendre conditions are trivially satisfied for (3.14) as \( dJ/dt' \) and \( dJ/dp' = 0 \) for all \( \tau \). QED

Theorem 3.2 shows that R&D production of the complete, marketable item adds computational but not substantive complexity to the standard optimal contracting outcomes.

3.4 R&D Procurement of a Subcomponent from a Profit Seeking Agent

In this subsection I analyze subcomponent production for the general model illustrated in Figure 3.3. The nomenclature used is the same as in subsection 3.3 except that subcomponent production rather than complete production is modeled; namely, \( U = U[S(\tau, p(\tau)), \Pi] \).

Agent's Problem [same as (3.13) is subsection 3.3]:

Maximize \( V \) w.r.t. Research cost: \( \frac{d}{d\lambda} \left\{ \int [v(t - p) - \psi(\lambda)] f d\tau \right\} = 0 \rightarrow \int vf, d\tau - \psi' = 0 \)

Principal's Agency Constrained Problem:

\[
J = \int U[S(\tau, p(\tau)), \Pi] f d\tau + \gamma \int \left[ v - \psi \right] f d\tau + \mu \left[ \int vf, d\tau - \psi' \right] + \delta \lambda
\]

The necessary condition for an optimum in \( \lambda \):

\[
\frac{dJ}{d\lambda} = \int U[S(\tau, p(\tau)), \Pi] f d\tau + \mu \left[ \int vf, d\tau - \psi' \right] = -\delta
\]

Integrating by parts (3.27) becomes

\[
- \int F_\lambda [S_p U_5 + p'S_p U_5 - t'U_\pi] d\tau + \mu \left[ \int F_\lambda v'(t' - p') d\tau + \psi'' \right] = -\delta
\]

The necessary conditions for optimal R&D price and Development cost functions are:

\[
\frac{dJ}{dt} = \frac{U_\pi}{v'} = \gamma + \mu \frac{f_\lambda}{f} \quad \forall \tau
\]

\[
\frac{dJ}{dp} = \frac{S_p U_5}{v'} = \gamma + \mu \frac{f_\lambda}{f} \quad \forall \tau
\]

Combining (3.29) and (3.30) yields

\[
S_p = \frac{U_\pi}{U_5} \quad \forall \tau \quad \left[ v_s S_p = 1 \text{ in subsection 3.3} \right]
\]

Differentiating (3.31) with respect to \( \tau \) yields

\[
p' = \frac{S_p U_5 + S_p S_p U_{55} - S_s U_{5\tau} + \tau (U_{5\pi - S_p U_{5s}})}{S_p U_{5\tau} - S_p U_5 - S_p^2 U_{55}} \quad \forall \tau
\]
Differentiating (3.29) with respect to \( \tau \) yields
\[
v'[(S_r + pS_p)U_{Sr} - t'U_{\pi \pi}] - v''(t' - p')U_{\pi} = \mu v^2 \frac{d}{dt}\left( \frac{f_\lambda}{f} \right) \quad \forall \tau
\]
3.33
Rearranging (3.33), we get
\[
p'v'S_p U_{Sr} + v''U_{\pi} - t'[v'U_{\pi \pi} + v''U_{\pi}'] = \mu v^2 \frac{d}{dt}\left( \frac{f_\lambda}{f} \right) - v'S_r U_{Sr}
\]
3.34
(3.32) and (3.34) can be placed in matrix form as
\[
\begin{bmatrix}
1 & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
p' \\
t'
\end{bmatrix} =
\begin{bmatrix}
E \\
F
\end{bmatrix}
\]
where
\[
B = \left. \frac{[U_{\pi \pi} - S_p U_{Sr}]}{[S_p U_{Sr} - S_{pp} U_p - S_p^2 U_{SS}]} \right| \geq 0, \quad E = \frac{S_{pp} U_p + S_p S_p U_{SS} - S_p U_{Sr}}{[S_p U_{Sr} - S_{pp} U_p - S_p^2 U_{SS}]}
\]
\[
C = vS_p U_{Sr} + v''U_{\pi}, \quad D = -[v'U_{\pi \pi} + v''U_{\pi}'] \geq 0, \quad \text{and} \quad F = \mu v^2 \frac{d}{dt}\left( \frac{f_\lambda}{f} \right) - v'SU_{Sr}
\]
Recalling that net compensation \((t - p)\) is the critical contractual concern, applications of Cramer's Rule and rearrangement yield
\[
t' - p' = \frac{\mu v^2 \frac{d}{dt}(f_\lambda)}{v'} - v' \left( v'[S_{pp} U_p + S_p U_{Sr} + S_p U_{SS}] + S_p U_p (S_p U_{Sr} - U_{\pi \pi}) \right) \quad \forall \tau
\]
3.35
where \( \Omega = 2S_p U_{Sr} - S_{pp} U_p - S_p^2 U_{SS} - U_{\pi \pi} \geq 0 \)

In subsection 3.3, positive monotonicity of the R&D contract (3.24) was a sufficient condition for maximization of the agent's concerns, which combined with the Fundamental Management Postulate (FMP) allowed Theorem 3.2 to be proven. The effect of subcomponent production prevents (3.35) from being signed a-priori.\(^8\) The FMP may be invoked here to assure that some sufficient condition for the principal's concerns holds [e.g., \((t' - p') \leq S_r U_5 / U_{\pi} \quad \forall \tau \) from (3.28) and (3.31)]; however, the agent's concerns [e.g., \((t' - p') \geq 0 \quad \forall \tau \)] need not be fulfilled by (3.35). The lack of certain monotonicity in (3.35) is important and intuitive -- an R&D contract where agent compensation decreases over some range of possible Research outcomes allows the principal to shift expenditures to other subcomponents. Thus, the agent's desire for a contract increasing in the Research outcome may not be reconcilable with the principal's intent to 'balance' the subcomponent inputs to overall production. In such an eventuality, \( \lambda^* = 0 \), and no R&D effort on subcomponent S would be undertaken.

\(^8\)This is true even if one assumes risk neutrality for both principal and agent.
3.5 The Relevance of an Observable Research Outcome

Assume the outcome of interim production, \( \hat{\tau} \), is not observable by the principal. After Research is concluded, the agent states an outcome, \( \hat{\tau} - \tau - \varepsilon \); the principal provides \( t(\hat{\tau}) \); and the agent produces \( S(\tau, p^*) \). The extent of understatement (\( \varepsilon > 0 \)) is bound by the principal’s ability to enforce the outcome of subcomponent production -- actual production must equal that claimed:

\[
S(\tau, p^*) = S(\hat{\tau}, p(\hat{\tau}))
\]

Applying the implicit function theorem to (3.36), we know that there exists an optimal Development funding decision by the agent, \( p^*(\varepsilon) \), and that

\[
p^* = \frac{S_\tau + p^*S_p}{S_p}.
\]

We can now characterize misrepresentation when \( \tau \) is not observable to the principal:

\[
\ell = v(t(\tau - \varepsilon) - p^*(\tau - \varepsilon)) - \psi(\lambda) + \rho \varepsilon \quad \Rightarrow \quad \frac{d\ell}{d\varepsilon} = -v(t' - p'^*) = -\rho
\]

Substituting in the expression for \( p^* \) from (3.37) and rearranging:

\[
t' - p' = S_\tau + \frac{p}{v'}
\]

Note that if \( \rho > 0 \), then \( \varepsilon = 0 \), \( \hat{\tau} = \tau \), and the lack of an observable is irrelevant. The condition on \( (t' - p') \) for this follows directly from (3.38)

\[
t' - p' > \frac{S_\tau}{S_p}
\]

Theorem 3.3: Whether contracting for an R&D effort of the complete final good [subsection 3.3] or for a subcomponent [subsection 3.4], no interior optimal ICDR contract (i.e., a contract that includes R&D, \( \lambda > 0 \)) based on a stated but unobserved Research outcome is possible.

Proof: For Complete Production [subsection 3.3]

An optimization condition (3.20) in subsection 3.3 is \( S_p = 1 \) for all \( \tau \) thus, for \( \hat{\tau} = \tau \), (3.39) becomes \( t' - p' > \frac{S_\tau}{S_p} \), which over all \( \tau \) violates the necessary condition (3.17) for an optimal \( \lambda \).

For Subcomponent Production [subsection 3.4]

Posit that for some \( \tau \), the agent states the truth, \( \hat{\tau} = \tau \). For this \( \tau \), an optimal compensation function \( [t(\tau) - p(\tau)] \) has the property \( (t' - p') \leq \frac{S_u}{U_\pi} \); i.e., the principal’s utility is greater than it would have been at \( \tau - \partial \tau \). Therefore, at this \( \tau \), by (3.39), \( \frac{S_u}{U_\pi} > \frac{S_p}{S_p} \Rightarrow S_p > \frac{U_{\pi}}{U_S} \). Were there no agency concerns, the principal would determine optimum \( p(\tau) \) by maximizing \( U[S(t, p(\tau)), B - p(\tau)] \) which results in the condition that \( S_p = \frac{U_{\pi}}{U_S} \). Any Cost of Agency will cause \( S_p < \frac{U_{\pi}}{U_S} \). This contradicts the assumption that the compensation function \( [t(\tau) - p(\tau)] \) was an optimal contract. This contradiction holds over all \( \tau \). QED
If the principal cannot observe the Research outcome, an optimum incentive compatible contract must be conditioned on the final production outcome. This is analogous to stating that in the absence of an observable Research outcome, no CES R&D contract can be optimal -- an optimal R&D contract with a profit seeking agent, must be of a FPPB form. An FPPB contracting environment can be illustrated by collapsing period 4 into period 3 of Figure 3.3 and proceeding with simplified analyses for complete and subcomponent production. In the case of complete production, the FPPB contract is the result of the standard moral hazard literature (e.g., Holmström). For subcomponent production, the principal's balancing motive again prevents certain monotonicity and \( \lambda^* > 0 \) cannot be proven (see Polk, 1993, pages 35 - 40).

4.0 R&D PROCUREMENT FROM A PERFORMANCE SEEKING AGENT

Several approaches to R&D production organization justify positing a type of agent who is motivated by the performance of the subcomponent he is contracted to innovate:

A manager in a large firm or government research lab who is directly responsible for an internal R&D effort of a subcomponent to some larger task.

A scientist agent under an external procurement such as those let for government funded projects or Industry/University collaborative efforts.

Often, such agents are contracted under the condition that they not profit from the work (beyond an agreed salary). This condition may be acceptable if the career impact of success is substantial.

Figure 4.1 illustrates R&D contracting to a performance seeking agent. The organization of production differs from that of Figure 3.3:

This type of agent may have limited access to capital markets; thus, initial funds, \( b \), may be provided to the agent as a Research price.

The cost of Development, \( t \), is transferred to the agent with no additional compensation; thus, \( t \) is also the price of Development.

The only source of monetary compensation for the agent is \( (b - \lambda) \).

Disutility of forgoing compensation, or of providing independent funding, \( (b - \lambda) < b \), is captured by \( \eta(\lambda - b) \) in the utility function, \( V \).

The information asymmetry is unchanged from section 3; the Research outcome, \( \tau \), is observed by both principal and agent, but the

![Figure 4.1. Subcomponent R&D Production with a Performance Seeking Agent](image-url)
inputs to Research, \( \lambda \) and \( \theta \), are observed only by the agent. The agent may choose to take advantage of the moral hazard inherent in the Research portion of the task by reporting a Research cost of \( \lambda^* < \lambda \) while having actually expended \( \lambda^* < \lambda \) and retaining \( (b - \lambda^*) > (b - \lambda) \) for himself.

The nomenclature and functional assumptions for the performance seeking agent are very similar to subsection 3.4 with the exclusion of \( p(t) \) and with the necessary modifications to the agent's utility; \( V = v[S(t, t)] + \eta(\lambda - b) \), where \( \nu' > 0, \nu'' < 0, \eta' < 0, \eta'' < 0 \). Analysis proceeds in two sequential steps: Full Information and Incomplete Information.

### 4.1 R&D Subcomponent Procurement under Full Information

#### The agent’s Participation Constraint:

\[
\int [v[S(\tau, t(\tau))] + \eta(\lambda - b)]f(\tau, \lambda) d\tau \geq \kappa
\]

#### The Principal's problem may then be formed as:

\[
b, \lambda, t(\tau) = \int U[S(\tau, t(\tau)), B - b - t(\tau)f(\tau, \lambda)] d\tau + \gamma \left[ \int [v + \eta]f(\tau, \lambda) d\tau - \kappa \right] + \xi b + \rho \lambda
\]

The first order conditions are:

\[
\frac{dj}{db} = -\int U_\pi f d\tau - \eta' + \xi = 0
\]

\[
\frac{dj}{d\lambda} = \int U_\lambda f d\tau + \gamma \left[ \int v f_\lambda d\tau + \eta' \right] + \rho = 0
\]

Pointwise optimization of \( J \) relative to \( t \) yields:

\[
\frac{S_t U_S - U_\pi}{S_\lambda v'} = -\gamma \quad \forall \tau
\]

**Lemma 4.1:** \( U_{\pi} - S_t U_S > 0 \) for all \( \tau \).

**Proof:** Immediate from \( \gamma > 0 \). QED

The third first order condition (4.5) can be used to obtain the first derivative of the optimal contract by differentiating each side relative to \( \tau \).

\[
t' = -\frac{S_t S_{\lambda} [v(S_{t} U_{SS} - U_{Sr}) - v''(S_{t} U_{S} - U_{r})] + v'S_{t} U_{\pi}}{S_t^2 [v'(S_{t} U_{SS} - U_{Sr}) - v'(S_{t} U_{S} - U_{r})] - v'S_{t} (S_{t} U_{5\pi} - U_{\pi}) + v'S_{t} U_{\pi}}
\]

Note that \( t' \) may be \( <= 0 \).
Lemma 4.2: The contract under complete information has the property \( t' > - \frac{S_t}{S_i} \forall \tau \).

Proof: see Appendix:

Lemma 4.3: The contract under complete information has the property \( t' \leq - \frac{S_{\tau}}{S_{n}} \forall \tau \).

Proof: see Appendix

Theorem 4.1: If \( S(\tau, t) \) is separable in its arguments (i.e., \( S_{\tau} = 0 \)), then the contract under complete information has the property that \( t' \leq 0 \) for all \( \tau \).

Proof: Immediate from Lemmas 4.2 and 4.3. QED

Assumption 4.1: The optimal Development transfer, \( t^* \), has the property \( t^* < \frac{S_{i}U_{\tau}}{U_{\pi} - S_{i}U_{\pi}} \forall \tau \).

[an overly strong analytical statement of the Fundamental Management Postulate].

Theorem 4.2: Given Lemma 4.2 and Assumption 4.1, the contract under complete information has the property that for each increase in \( \tau \), both the principal and the agent are made better off. This property is captured by the following inequality:

\[-\frac{S_{i}}{S_{i}} < t' < \frac{S_{i}U_{\tau}}{U_{\pi} - S_{i}U_{\pi}} \forall \tau \]

Proof: That the inequality implies both would be better off over all \( d\tau \) is shown

\[
\frac{dS}{d\tau} = S_{i} + tS_{i} \implies t' \geq -\frac{S_{i}}{S_{i}} \implies \frac{dS}{d\tau} > 0
\]

\[
\frac{dU}{d\tau} = S_{i}U_{\tau} - t'(U_{\pi} - S_{i}U_{\pi}) \implies t' < \frac{S_{i}U_{\tau}}{U_{\pi} - S_{i}U_{\pi}} \implies \frac{dU}{d\tau} > 0. \text{ QED}
\]

Corollary 4.1: If the relative risk aversion condition \(-\frac{v''}{v'} \geq \frac{S_{i}}{S_{i}S_{\tau}} + \frac{\Omega}{S_{i}U_{\pi}(U_{\pi} - S_{i}U_{\pi})} \forall \tau\),

where \( \Omega = S_{i}U_{\tau}U_{\pi} - S_{i}U_{\pi}(S_{i}U_{\pi} - U_{\pi\pi}) + S_{i}U_{\pi}(S_{i}U_{\tau\tau} - U_{\tau\tau}) \leq 0 \forall \tau \), holds, then Assumption 4.1 is satisfied and Theorem 4.2 holds:

Proof: see Appendix

Corollary 4.2: If \( U_{\tau\tau}, U_{\pi\pi}, U_{\pi\tau}, \) and \( v'' = 0 \) [Tan's Assumptions], then

i. \( t' = -\frac{S_{i}}{S_{n}} \forall \tau \geq 0 \)
\[
\text{ii. } \frac{d}{d\tau}[U_\pi - S_\tau U_\beta] = \frac{d}{d\tau}[S_\tau \psi'] = 0 \quad \forall \tau
\]

iii. Assumption 4.1 holds, implying that Theorem 4.2 and Corollary 4.1 hold.

Proof: see Appendix

Corollary 4.2 provides a useful comparison to the existing R&D contracting literature when one adds the element of an agent motivated by the performance capability of his subcomponent. Under conditions of optimum risk sharing, the optimum contract has the standard properties of positive monotonicity in the observable and uniformly increasing utilities for principal and agent with each increase in the observable. These results suggest that the new type of agency motivation introduced here does not, in and of itself, fundamentally affect contracting.

Comparing the agency model in section 4 to that in section 3, recall that with subcomponent production (subsection 3.4), optimal risk sharing under full information, \( \mu = 0 \), did not necessarily result in both the principal and agent being better off over all levels of output.\(^9\) Further, it could not definitely be shown in subsection 3.4 that either the full or incomplete information solution resulted in the principal or agent being better off in expectation. Through Theorem 4.2, Corollary 4.1 and Corollary 4.2, I have shown conditions under which the R&D model of section 4 exhibits the standard qualities of optimal risk sharing under full information. By Theorem 4.3, I establish that both principal and agent are generally better off in expectation under full information for the model examined in section 4.

**Theorem 4.3:** Any positive R&D result (\( \lambda > 0 \)) from the full information first order conditions (4.3, 4.4, and 4.5) implies that both principal and agent are better off in expectation.

Proof: see Appendix

**4.2 R&D Subcomponent Procurement under Incomplete Information**

In this section, the information asymmetry illustrated in Figure 4.1 and common to the analyses in section 3 is considered for the performance seeking agent. This asymmetry should result in a moral hazard agency concern as the subcomponent performance interests of the agent do not perfectly overlap the broader final good performance interests of the principal.

*The Agent's problem is to choose \( \lambda \) to maximize:*

\[
\ell_1 = \int \left[ \psi(S(\tau, t(\tau))) + \eta(\lambda - b) \psi(\tau, \lambda) \right] d\tau
\]  \hspace{1cm} (4.7)

\(^9\)Principal and agent were shown to be better off over all \( \tau \) with complete production (subsection 3.3).
\[
\frac{df_1}{d\lambda} : \int [(v + \eta)f_\lambda + \eta'f] d\tau = \int v f_\lambda d\tau + \eta' = 0 \quad 4.8
\]

*The Agency Constrained Principal's problem:*

\[
\begin{align*} 
\int_{b,t,\lambda(t)} & = \int U[S, \pi\lambda(\tau, \lambda)]d\tau + \gamma \left[ \int [v + \eta\pi\lambda(\tau, \lambda)]d\tau - K \right] + \mu \left[ \int v f_\lambda d\tau + \eta' \right] + \xi B + \rho \lambda 
\end{align*} 
\]

*Participation Constraint  Incentive Compatibility Constraint  4.9*

First Order Conditions

\[
\frac{dJ}{db} : - \int U_{t} f d\tau - \gamma \eta' - \mu \eta'' + \xi = 0 
\]

\[
\frac{dJ}{d\lambda} : \int U f_\lambda d\tau + \mu \left[ \int v f_\lambda d\tau + \eta'' \right] + \rho = 0 
\]

4.10

4.11

Integration of (4.11) by parts yields:

\[
- \int F_\lambda [S_{\tau} U_{S} - t' (U_{\pi} - S_{\tau} U_{S})] d\tau + \mu \left[ \eta'' - \int F_{\lambda} v'[S_{\tau} + t' S_{\tau}] d\tau \right] + \rho = 0 
\]

4.12

Pointwise optimization of J relative to t yields:

\[
\frac{S_{\tau} U_{S} - U_{\pi}}{S_{\tau} v'} = - \gamma - \mu \frac{f_\lambda}{f} \quad \forall \tau 
\]

4.13

Differentiating (4.13) with respect to \( \tau \) yields the expression for \( t' \):

\[
t' = - \frac{S_{\tau} S_{\tau} \left[ v'(S_{\tau} U_{S} - U_{\pi}) - v''(S_{\tau} U_{S} - U_{\pi}) \right] + v' S_{\tau} U_{\pi} + \mu (v' S_{\tau}^2 \frac{\partial}{\partial \tau} \left( \frac{f_\lambda}{f} \right))}{S_{\tau}^2 \left[ v'(S_{\tau} U_{S} - U_{\pi}) - v''(S_{\tau} U_{S} - U_{\pi}) \right] + v' S_{\tau} (S_{\tau} U_{S} - U_{\pi}) + v' S_{\tau} U_{\pi}} 
\]

4.14

Note that \( t' \) may be \( \leq 0 \).

In order to prove that these first order conditions describe the optimal solution to the R&D problem illustrated in Figure 4.1 and in order to characterize the incentive compatible contract \( t(\tau) \), a series of lemmas and theorems follows:

**Lemma 4.4:** As with full information, \( S_{\tau} U_{S} - U_{\pi} \leq 0 \) for all \( \tau \).

**Proof:** see Appendix

**Theorem 4.4:** \( \mu > 0 \); i.e., there is a moral hazard agency contracting concern.

**Proof:** see Appendix

**Lemma 4.5:** Given Lemma 4.2 and Theorem 4.4, \( t' > \frac{S_{\tau}}{S_{\tau}} \quad \forall \tau \) \quad [a sufficient condition to characterize the solution of the agent's problem (4.8)].

**Proof:** see Appendix
For subcomponent R&D production under profit seeking agency (subsection 3.4) we were unable to show that the contract resulting from the first order conditions had the property that the agent was better off for each differential increase in the Research outcome. This failing prohibited the establishment of a theorem characterizing optimal subcomponent R&D contracting with a profit seeking agent. Under performance seeking agency we have proven Lemma 4.5 which establishes that the agent is better off for each differential increase in the Research outcome. This result plays a pivotal role in the theorems to follow as it is an overly strict sufficient condition for the agent’s concerns. The Fundamental Management Postulate (FMP) allows us to assume that a sufficient condition for the principal’s concerns is satisfied; however, for ease of exposition we will assume that the FMP is strengthened and allows for a condition on the principal’s problem analogous to Lemma 4.5.

**Assumption 4.2:** $\tau' < \frac{S_t U_5}{U_{\pi} - S_t U_5}$ for all $\tau$. [The principal is better off for each $\tau$]

**Theorem 4.5:** Given Assumption 4.2, the solution $\{b^*, \lambda^*, t^*(\tau)\}$ derived from the first order necessary conditions to the principal’s problem (4.9) is the unique optimum solution and results in positive research investment [$\lambda^* > 0$].

**Proof:**

i. From (4.12) it is clear that Lemma 4.5 provides a sufficient condition for the agent’s optimization with respect to $\lambda$.

ii. Assumption 4.2 is a sufficient condition for concavity with respect to $\lambda$ of the maximand in the principal’s optimization problem:

The second derivative of the maximand combined with Assumption 4.2 yields

$$-\int F_{\lambda \lambda}[S_t U_5 - t'(U_{\pi} - S_t U_5)]d\tau \leq 0.$$  

ii. The principal’s side of (4.10) is quasi-concave, the agent’s convex:

The maximum will occur in the interior.  QED

**Corollary 4.3:** Assumption 4.2 and its role in Theorem 4.5 can be characterized by the relative risk aversion condition [Analogous to Corollary 4.1]:

$$-\frac{v''}{v} \geq \frac{S_{tt}}{S_t S_{t}} + \frac{v S_t}{S_t U_{\pi}} \frac{d}{d\tau} \left( f_{\lambda} \right) + \frac{\Omega}{S_t U_{\pi} (U_{\pi} - S_t U_5)} \forall \tau$$

where $\Omega = S_t U_5 U_{\pi} - S_t U_5 (S_t U_{5\pi} - U_{5\pi}) + S_t U_{\pi} (S_t U_{55} - U_{55}) \leq 0 \forall \tau$.

**Proof:** Immediate with the same approach as Corollary 4.1.  QED
4.3 The Relevance of an Observable Research Outcome

Assume that \( \tau \) is not observable. Instead after Research is concluded, the agent states an outcome, \( \hat{\tau} - \tau - \varepsilon \); the principal provides the transfer \( t(\hat{\tau}) \); and the agent produces \( S(\tau, t^*) \). In addressing this interim adverse selection problem, the critical issue, as in subsection 3.5, is the enforcement of the final outcome. Definition of Enforcement:

\[
S(\tau, t^*) = S(\hat{\tau}, t(\hat{\tau}))
\]

By the implicit function theorem, we know that there exists a \( t^*(\varepsilon) \) and that

\[
t^* = \frac{S_\tau + t^* S_t}{S_t}.
\]

We can characterize misrepresentation when \( \tau \) is not observable by maximizing the monetary gain from Development, referred to colloquially as skimming:

\[
\ell = v[S(\tau, t^*(\tau - \varepsilon))] + \eta[t^*(\tau - \varepsilon) - t(\tau - \varepsilon)] + \rho \varepsilon
\]

\[
\frac{d\ell}{d\varepsilon} = -t^* v S_t - t^* \eta' + t' \eta' = -\rho = t' \eta' - t^* [v S_t + \eta']
\]

Substituting in the expression for \( t^* \) from the enforcement condition and rearranging:

\[
t' = \frac{S_\tau}{S_t} \left[ 1 + \frac{\eta'}{v S_t} \right] + \rho
\]

Note that if \( \rho > 0 \), then \( \varepsilon = 0, \hat{\tau} = \tau \), and the lack of an observable is irrelevant; no skimming takes place. The condition on \( t' \) for this follows directly from (4.17):

\[
t' > \frac{S_\tau}{S_t} \left[ 1 + \frac{\eta'}{v S_t} \right], \text{ where we set } \Gamma = \left[ 1 + \frac{\eta'}{v S_t} \right] \text{ and note that } \Gamma \leq 1
\]

From Lemma 4.5 we know that \( t' > \frac{S_\tau}{S_t} \); thus, \( \Gamma \) determines the relevance of the observable and can be thought of as \( 1 + \frac{\text{Marginal Disutility of Revealing } \tau}{\text{Marginal Utility of Revealing } \tau} \). Therefore, \( \Gamma \) represents an agent attribute that affects whether or not interim adverse selection binds.

**Theorem 4.6:** When contracting with a performance seeking agent with a Research outcome unobserved by the principal, the candidate optimal contract from the first order conditions [4.10, 4.11, and 4.13] is the optimal ICDR R&D contract if, for all Research outcomes, the agent's marginal utility from skimming Development funds, \(-\eta'\), is less than or equal to his marginal utility from pursuing subcomponent performance, \(v S_t\).
Proof: Posit a candidate optimal \( \hat{t}(\tau) \) such that Assumption 4.2 holds; i.e., \( \hat{t}' < \frac{S_t U_S}{U_{\pi} - S_t U_S} \) at a particular \( \tau \). If \( \hat{t}(\tau) \) is such that at this same \( \tau \) condition (4.18) is satisfied, then we know that

\[
\frac{S_t U_S}{U_{\pi} - S_t U_S} > \frac{S_t}{S_t} \left[ 1 + \frac{\eta'}{v'S_t} \right] \Rightarrow -\frac{\eta'}{v'S_t} < \frac{U_{\pi}}{U_{\pi} - S_t U_S}
\]

By Lemma 4.4 \((U_{\pi} - S_t U_S \geq 0)\), we know that if \(-\frac{\eta'}{v'S_t} < 1\), then \(\hat{t} = \tau\). From Theorem 4.5, we know that one possible optimum incentive compatible contract has the property that Assumption 4.2 holds for all \( \tau \); thus, as long as \(-\frac{\eta'}{v'S_t} < 1 \) \( \forall \tau \) for this optimum contract, then the agent will state the true outcome of Research, \( \tau \), when \( \tau \) is unobservable. QED

Corollary 4.4: An optimal ICDR R&D contract that meets the requirements of Theorem 4.6 may be weakly negatively monotonic.

Proof: Note that condition (4.18) is the lower bound on \( t' \) if no skimming is to occur. Theorem 4.6 requires \(-\eta' \leq v'S_t \) for all \( \tau \). Therefore, the maximum lower bound for \( t' \) is \( t' = 0 \) for all \( \tau \). QED

When the requirements for Theorem 4.6 do not hold for the candidate optimal ICDR R&D contract, then interim adverse selection may bind resulting in the agent under stating the Research outcome and skimming some percentage of Development funding. The principal’s task of procuring all other subcomponents to the final good is facilitated by knowing the true Research outcome of subcomponent \( S \), else expenditure on the other subcomponents will likely be suboptimal. Thus, a candidate optimal contract that is insufficiently steep over some range of possible Research outcomes must be modified so that the true outcome of Research is reported even if some skimming occurs. This is the adverse selection problem considered by Guesnerie and Laffont and can be similarly handled by modifying the candidate optimal contract with flat segments.\(^{10}\) Over these flattened segments the principal accepts monetary skimming in exchange for the accurate revelation of the Research outcome. In the extreme, the candidate optimal would be transformed into a Pure Fixed Price (PFP) contract, with a fixed sum transferred to the agent who is then allowed to retain whatever percentage he wishes and use the rest to pursue subcomponent performance. Note that only with a performance seeking agent would such an arrangement make any sense to the principal.

5.0 PROCUREMENT POLICY IMPLICATIONS

The potential benefits of innovation motivate R&D efforts throughout the economy. Within the production organization of a complex final good, a principal exists who must manage the procurement and integration of the subcomponents that constitute the final good. The agency implications of the heightened uncertainties that distinguish R&D from standard production should influence which subcomponents undergo R&D and what contractual forms are used. Through analyses of various agency and information structures, this research has characterized optimal R&D contracting, enabling significant insight into private and public sector procurement.

Both principal and agent could benefit from contracts where subcomponent price and performance capability vary with the Research outcome — the principal by balancing the capabilities of all subcomponents so as to minimize unnecessary overall costs and the agent by sharing the cost exposure of the risky Research endeavor. Ideally, the principal and agent would agree to a specific Research investment and an explicit range of Development efforts contingent on the outcome of the Research. I have described the appropriate Real World analog of this ideal contract as a Cost Exposure Sharing (CES) contract. Only when agency effects prevent the existence of CES contracts would the principal's contracting preferences default to Fixed Price Performance Based (FPPB) or Pure Fixed Price (PFP) contracts.

Table 5.1 summarizes the results of this research. Depending on agent motivation and the principal's knowledge of the Research outcome, the preferred CES contract may not be attainable. Most striking, if the principal cannot observe the Research outcome from a profit seeking agent, then a contract dependent on the Research outcome cannot be devised; requiring the use of a FPPB contract. The efficacy of profit seeking agency is brought into general question by the inability to show that R&D contracts acceptable to the principal would necessarily be acceptable to the agent. The implications of a limited choice set for R&D bodes ill for a principal whose performance seeking agents (e.g., internal divisions) may then seek additional rents.

The following subsections examine how the interplay of observability and agency motivation affect private sector and public sector R&D procurement. Not surprisingly, the effects are different due to the disparate forces shaping each sector.

Table 5.1. Optimal R&D Subcomponent Contracts for different Motivation and Information Mixes

<table>
<thead>
<tr>
<th>Principal's Knowledge of the Research Outcome</th>
<th>Profit Seeking</th>
<th>(&lt;\text{Subsection 3.4})</th>
<th>FPPB*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observable</td>
<td>CES*</td>
<td>Theorem 3.3</td>
<td></td>
</tr>
<tr>
<td>Unobservable</td>
<td>Agent Motivation Type</td>
<td>CES [preferred]</td>
<td></td>
</tr>
<tr>
<td>Performance Seeking</td>
<td>CES</td>
<td>Theorem 4.5</td>
<td>PFP</td>
</tr>
<tr>
<td></td>
<td>Theorem 4.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

R&D not assured, may revert to Off-the-Shelf
5.1 PRIVATE SECTOR PROCUREMENT IMPLICATIONS

Product innovation is critical for a firm competing in a free market economy -- the threat of eclipse by existing or new competitors places a premium on cost efficient R&D.\textsuperscript{11} Thus, a firm's subcomponent R&D procurement plans should depend on the efficacy of the various agency choices available. Figures 5.1 and 5.2 illustrate theoretically consistent, hypothetical contracting options for subcomponent R&D contracted to a profit seeking agent. In Figure 5.1, the principal, by knowing the Research outcome, is able to balance the performance contribution of the subcomponent [i.e., $S(r, p(r))]$ with its budget impact on all other concurrent subcomponent decisions [i.e., $t(r)$]. This concurrent balancing ability is lost when the Research outcome is unobservable and the contract must revert to a FPPB contract as in Figure 5.2. The expected price per unit performance from the FPPB contract of Figure 5.2 might be greater or less than that from the CES contract of Figure 5.1; however, it is clear that the value of certainty over concurrent subcomponent balancing is lost with the FPPB form.

![Figure 5.1. CES Subcomponent R&D Contract with a Profit Seeking Agent](image1)

![Figure 5.2. FPPB Subcomponent R&D Contract with a Profit Seeking Agent](image2)

The overall value loss to the principal from balance uncertainty can come from either the time value of waiting for subcomponent $S$ to be fully produced or from the increased risks of finalizing all other subcomponent decisions based on an expected value for $S$. In either case, the loss from unobservability is as valid a justification as subcomponent price to entice the principal to utilize a performance seeking agent (e.g., an internal division of the principal's firm). Figure 5.3 provides a performance seeking alternative to the profit seeking contracts of Figures 5.1 and 5.2.

\textsuperscript{11} The relevance and effectiveness of market forces are implicit here in the terminology "cost efficient R&D," as an R&D decision must be based on an expected value calculation made relative to the anticipated market demand for the innovated product.

23
Recall from section 4 that, if the research outcome is observable, a performance seeking agent can only receive financial recompense from the difference between the Research transfer, b, and his Research investment decision, \( \lambda \). If the Research outcome is unobservable and his marginal utility of \textit{skimming} is high enough over some potential values of the Research outcome, then \textit{skimming} becomes a potential second source of financial recompense and the CES contract must be modified with PFP segments to partially retain the desired balancing property.

In deciding with what type of agent to contract, the principal must consider the combined expected impacts of subcomponent price and balancing capability. When the Research outcome is unobservable and the ability to concurrently balance decisions across subcomponents is very important,\(^{12}\) the value to the principal of a performance seeking agent will be high. Without choice among performance seeking agents, an internal division of the principal's firm can exact a heightened rent (larger transfer b). In such cases, the principal should be interested in stimulating external performance seeking agency. A number of recent R&D efforts, the principals of which are arguably subject to high balancing valuations, illustrate a mechanism that may effectively transform profit seekers into performance seekers:

Much of the first $1 billion for the Iridium global cellular system, has been invested by the principal, Motorola, and critical R&D subcomponent suppliers.

The approximately $5 billion required to produce the first 777 jet liner was provided by Boeing and a group of critical subcomponent suppliers who are guaranteed long-term supply contracts, the volume of which depends on the market success of the 777.

Having agents pay for the right to share in any returns from the final product may allow a principal to avoid the rents of an internal agent, and, thus, bring an innovated product to market at a more competitive price. Analysis of such a contract is beyond the model presented here as the agent's revenue becomes dependent on the final good as well as his subcomponent.

\(^{12}\)e.g., high potential returns to technology, long Development phases or the threat of rapid innovation by a competitor, and large overall expenditures to bring a product to market
5.2 Public Sector Procurement Implications

Agencies of the U.S. government regularly manage substantial R&D efforts. Numerous next generation weapon systems and the national space exploration effort are examples of post-WWII publicly procured complex products that have required subcomponent R&D. The public sector principal who manages procurement and integration of the subcomponents should have a valuation over the final product that is essentially equivalent to the analogous private sector principal; namely, to balance the performance contribution of a single subcomponent against that subcomponent’s budgetary impact on the other required subcomponents. What operationally distinguishes the public sector principal from the private is a vast internal set of agents interacting with a system of federal procurement regulations that discourages external R&D agency. A little history serves as an appropriate precursor for explaining this distinction.

Prior to WWII, a standard U.S. government subcomponent R&D contract was the FPPB contract with profit seeking agents. For instance, innovations to the U.S. Army Air Corps (USAAC) fighting capability were procured by designating performance criteria for a desired plane and a willingness to purchase certain volumes of production at certain prices for planes that met or exceeded those criteria. Private firms raised the risk capital to perform the required R&D and then entered their prototypes in Fly-Off competitions, from which the USAAC decided which entrants should be procured at what volumes. At the outset of hostilities, the extent and pace of R&D deemed necessary by the USAAC resulted in the almost uniform adoption of Cost-Plus contacting, in which a firm received progress payments that covered all costs incurred in Research, Development, and subsequent production. Based on the eventual outcome of the R&D, the firm would receive an additional fee. This fee might be solely dependent on physical performance or it might depend on a number of parameters valued by the principal, such as date of delivery and/or final cost vs. bid price.13

The exigencies of WWII motivated an immense expansion of R&D efforts conducted within agencies of the government, best exemplified by the growth of National Laboratories, such as those that grew out of the Manhattan Project. During the Cold War, the National Laboratories became the principals in charge of designing, procuring, and, often, integrating the subcomponents to public R&D efforts. Substantial subcomponent R&D was contracted internally, to subdivisions of the National Laboratories, while the standard external R&D contract to profit seeking firms remained Cost-Plus in form.

For a profit seeking agent, the Cost-Plus contract can be a special case of either the CES or the FPPB contract. In either case, the critical feature imposed by the Cost-Plus restriction is that

---

the firm's costs must not exceed the price paid by the principal. With reference to Figure 5.1, this is equivalent to the statement that net compensation must be non-negative over all potential Research outcomes. The following simple and reasonable example illustrates the impact of a non-negative profit restriction on a profit seeking agent's R&D decisions:

Given a 2-state R&D environment \( \{ \tau = 1 - \rho, \tau_o = \rho \} \) and assuming that the Research outcome is observable, the goal is to determine and employ a CES contract \( \{ \lambda, p^*(\tau), t^*(\tau) \} \) such that an optimal R&D effort is undertaken.

**Assumption 5.1:** The Development cost for the profit seeking agent if the Research outcome is unsuccessful is at least as large as the *Off-the-Shelf* price; namely, \( p(\tau_o) \geq P_{S_o} \).

Recalling the profit seeking agent's utility function, \( V = v(t - p) - \psi(\lambda) \), the following inequality must hold if any Research effort \( (\lambda > 0) \) is to be undertaken

\[
\rho \left[ v(t(\tau_o) - p(\tau_o)) \right] + (1 - \rho) \left[ v(t(\bar{\tau}) - p(\bar{\tau})) \right] > v(t(\tau_o) - P_{S_o}). \tag{5.1}
\]

Applying Assumption 5.1 to condition (5.1) yields two additional inequalities

\[
v(t(\tau_o) - p(\tau_o)) \leq v(t(\bar{\tau}) - P_{S_o}) < v(t(\bar{\tau}) - p(\bar{\tau})), \tag{5.2}
\]

which directly imply that the profit from a successful Research outcome must increasingly exceed the *Off-the-Shelf* profit with increasing Research risk, \( \rho \), if any Research effort is to be undertaken. In this regard, the non-negative profit restriction of any Cost-Plus contracting structure implies that the profit for a successful Research outcome must become quite high as the Research risk of failure increases. Conditions (5.1) and (5.2) indicate that the impact of requiring Cost-Plus contracting for R&D procurements from profit seekers can only raise the price of external procurement, favoring internally conducted R&D.

The preceding example illustrates how restricting external procurements to Cost-Plus contracts should bias Federal Laboratories toward internal R&D subcomponent procurement. The application of one Federal Acquisition Regulation enshrines this bias for all high risk, avant-garde R&D -- Profit margins under Cost-Plus contracts are restricted by law to be no greater than 15\%.\(^{14}\) Thus, the government, having granted a Federal Laboratory the position of principal, so restricts the principal's contracting options that the principal may justly reserve the most technologically engaging R&D tasks for his internal agents. The internal agents support the status quo contracting practices as they are allowed to extract substantial rents. Further, the Federal Laboratories have traditionally operated as monopolies over subsets of publicly funded R&D; thus, the principals are in agreement with their internal agents on maintaining a contracting structure that justifies institutionalized innovation; surely an oxymoron.

\(^{14}\) *Federal Acquisition Regulation, Part 15, Section 15, 903, Paragraph D1.*
APPENDIX A: EX ANTE COMPLETE VS INCOMPLETE CONTRACTS

Within the context of the principals and agents considered by this research, I contend that it is appropriate to consider only complete contracting as the justifications for efficient incomplete contracts either do not apply or are irrelevant:

Tirole suggests that renegotiation between production stages may be justified by the principal not knowing his values *ex ante* for outcomes of agent investment -- perhaps because of the impacts of concurrent procurements. My analysis internalizes the impacts of concurrent procurements with the proxy of R&D-residual funds for other subcomponent procurements. The principal is *ex ante* aware of the tradeoffs between subcomponents even though he is not aware of the specific trades he will make.

Ching-To Albert Ma has shown that renegotiation based on concerns of the agent may be an irrelevant complication -- the optimal contract remains the *ex ante* complete contract.

Crocker and Reynolds posit that incomplete contracts may result from an intermediate principal's unwillingness to commit the effort to arrange complete contracts. Our research has concentrated on the level of principal responsible for integrating a final good for which he garners immediate monetary or property right value. Such a principal is motivated to pursue efficiency and should not be as eager to accept an agent's incomplete contract offer as is Crocker's and Reynolds's intermediate principal.

---


Lemma 4.2: The optimal contract under complete information has the property that \( t' > -\frac{S_t}{S_i} \) \( \forall \tau \).

Proof: From (4.6):

\[
t' = -\frac{S_t X + A}{S_i X + B}, \quad \text{where} \quad A = \nu S_i U_{\pi} > 0 \quad \text{and} \quad B = \nu \left[ S_i U_{\pi} - S_i (S_i U_{\pi} - U_{\pi}) \right] < 0
\]

\[
t' = -\frac{S_t X}{S_i X + B} - \frac{A}{S_i X + B} > -\frac{S_t X}{S_t X + B} > -\frac{S_t}{S_i} \quad \text{QED}
\]

Lemma 4.3: The contract under complete information has the property that \( t' \leq -\frac{S_t}{S_i} \) \( \forall \tau \).

Proof:

\[
t' = -\frac{Y + \nu S_i U_{\pi}}{C + \nu S_i U_{\pi}},
\]

where \( Y = S_i S_{\pi} \left[ \nu S_i (S_i U_{\pi} - U_{\pi}) - \nu''(S_i U_{\pi} - U_{\pi}) \right] < 0 \)

\[
C = S_i^2 \left[ \nu S_i (S_i U_{\pi} - U_{\pi}) - \nu''(S_i U_{\pi} - U_{\pi}) \right] - \nu S_i (S_i U_{\pi} - U_{\pi}) < 0
\]

\[
t' = -\frac{Y}{C + \nu S_i U_{\pi}} - \frac{\nu S_i U_{\pi}}{C + \nu S_i U_{\pi}} < -\frac{\nu S_i U_{\pi}}{C + \nu S_i U_{\pi}} < -\frac{S_t}{S_i} \quad \text{QED}
\]

Corollary 4.1: If the relative risk aversion condition \( -\frac{\nu''}{\nu'} \geq \frac{S_i}{S_{\pi}} + \frac{\Omega}{S_i U_{\pi} (U_{\pi} - S_i U_{\pi})} \) \( \forall \tau \) holds, where \( \Omega = S_{\pi} U_{\pi} U_{\pi} - S_i U_{\pi} (S_i U_{\pi} - U_{\pi}) + S_i U_{\pi} (S_i U_{\pi} - U_{\pi}) \leq 0 \) \( \forall \tau \), then Assumption 4.1 and Theorem 4.2 hold:

Proof: Follows directly by establishing the condition on \( t' \) such that the principal would be no better off with each increasing revelation in the observable \( \tau \).

\[
t' \bigg|_{\frac{4 U_{\pi}}{\nu'}} = \frac{S_i U_{\pi}}{U_{\pi} - S_i U_{\pi}}
\]

Set this as the maximum \( t' \) allowed relative to the candidate solution (4.6):

\[
\frac{S_i U_{\pi}}{U_{\pi} - S_i U_{\pi}} \geq \frac{S_i S_{\pi} \left[ \nu S_i (S_i U_{\pi} - U_{\pi}) - \nu''(S_i U_{\pi} - U_{\pi}) \right] + S_i U_{\pi}}{S_i^2 \left[ \nu S_i (S_i U_{\pi} - U_{\pi}) - \nu''(S_i U_{\pi} - U_{\pi}) \right] - \nu S_i (S_i U_{\pi} - U_{\pi}) + \nu S_i U_{\pi}} \quad \forall \tau
\]

Rearranging terms and separating out all \( \nu' \) and \( \nu'' \) terms yields:

\[
-\nu'' (U_{\pi} - S_i U_{\pi}) \left[ S_i S_{\pi} (U_{\pi} - S_i U_{\pi}) \right] \geq \nu \left[ S_i S_{\pi}^2 U_{\pi} (S_i U_{\pi} - U_{\pi}) - S_i S_{\pi} U_{\pi} (S_i U_{\pi} - U_{\pi}) + S_i S_{\pi} U_{\pi} \right] \quad \forall \tau
\]

Cancellation and rearrangement lead to the desired result: \( -\frac{\nu''}{\nu} \geq \frac{S_i}{S_{\pi} S_{\pi}} + \frac{\Omega}{S_i U_{\pi} (U_{\pi} - S_i U_{\pi})} \) \( \forall \tau \) QED
Corollary 4.2: If \( U_{ss}, U_{st}, U_{st}, \) and \( v^* = 0 \) [Tan's Assumptions], then

i. \( t' = \frac{S_{\tau}}{S_n} \forall \tau \)

ii. \( \frac{d}{dt}[U_s - S_tU_s] = \frac{d}{dt}[S_tv'] = 0 \forall \tau \)

iii. Assumption 4.1 holds, implying that Theorem 4.2 and Corollary 4.1 hold.

Proof:

i. immediate from (4.6) (maximum slope by Lemma 4.3)

ii. \( \frac{d}{dt}[U_s - S_tU_s] = t'[2S_tU_{st} - U_{ss} - S_tU_s - S_t^2U_{st}] + S_tU_{st} - S_tU_s - S_tS_tU_{st} = 0 \)

\( \frac{d}{dt}[S_tv'] = t'[v^*S_t^2 + v^*S_t] + v^*S_tS_t + v^*S_t = 0 \)

iii. immediate from (ii) and Lemma 4.1. QED

Theorem 4.3: Any positive R&D result \( (\lambda > 0) \) from the full information first order conditions (4.3, 4.4, and 4.5) implies that both principal and agent are better off in expectation.

Proof:

That the agent is better off over all \( \tau \) is provided by Lemma 4.2 and Theorem 4.2.

That the principal is better off for any \( \lambda > 0 \) is shown through examination of condition (4.4).

Recall (4.4): \( \frac{\partial J}{\partial \lambda} : \int U_{f_t}d\tau + \gamma \left[ \int vf_{f_t}d\tau + \eta' \right] + \rho = 0 \)

For \( \lambda > 0, \rho = 0 \) and the participation constraint (4.1) holds; therefore \( \gamma > 0 \).

Now define the Marginal R&D case (MR&D) as the solution to conditions (4.3, 4.4, and 4.5) that results in

\( \int U[S(\tau, t(\tau)), B - b - t(\tau)]d\tau = U(S_0, B - P_0) \)

\( \implies \int U_{f_t}d\tau \bigg|_{MR&D} = 0 \stackrel{(4.4)}{\implies} \left[ \int vf_{f_t}d\tau + \eta' \right] \bigg|_{MR&D} = 0 \)

Now examine the nature of MR&D for a differential increase in \( \lambda \):

\( \frac{\partial}{\partial \lambda} \left[ \int vf_{f_t}d\tau + \eta' \right] = \int vf_{f_t}d\tau + \eta'' = -\int F_{f_t}v(S_0 + t's_0)d\tau + \eta'' < 0 \)

Therefore, from (4.4), \( \int U_{f_t}d\tau > 0 \) for R&D efforts above MR&D.

Now assume that at MR&D the principal opts for \( U_0 \) rather than for \( E[U] \); therefore,

\( \int U_{f_t}d\tau > 0 \) for all R&D undertaken \( \implies -\int F_{f_t} \left( \frac{\partial U}{\partial \tau} \right) d\tau > 0, \)

which implies that the principal is better off in expectation for all R&D undertaken. QED
Lemma 4.4: In the incomplete information case, as with complete information, $\Sigma U_\tau - U_\tau \leq 0$ for all $\tau$.

Proof: Define $\Psi$ as the space of (S, II) pairs on which the principal's utility may be assessed. Also define $t^*$ as a candidate for the optimal contract from the first order conditions. For any true $t^*$, we know that

i. $\frac{\Sigma U_\tau - U_\tau}{\Sigma \lambda v'} = -\gamma - \mu \frac{f_\lambda}{f}$

ii. $\frac{f_\lambda}{f} \leftrightarrow 0$ depending on $\tau$

iii. $\Sigma U_\tau - U_\tau < 0$ for some $\tau$ regardless of $\mu \leftrightarrow 0$.

The proof proceeds by contradiction:

Assume that given $t^*$, for some $\tau$, $\Sigma U_\tau - U_\tau > 0$. Then from (iii) there is a point $B$ in $\Psi$ where $\Sigma U_\tau - U_\tau = 0$. Further, characterize $t^*$ by the intervals in $\Psi$ around $B$

over $[A, B]$ $t^*(\tau)$ s.t. $\Sigma U_\tau - U_\tau \leq 0$

over $(B, C]$ $t^*(\tau)$ s.t. $\Sigma U_\tau - U_\tau > 0$

Now consider $t^{**} = \begin{cases} t^* \text{ over } [A, B] \\ s.t. \Sigma U_\tau - U_\tau = 0 \text{ over } (B, C] \end{cases}$

Over $(B, C]$ note that both principal and agent are better off with $t^{**}$ then with $t^*$; therefore, $t^*$ cannot be the optimal solution. This is a contradiction, and thus

$\Sigma U_\tau - U_\tau \leq 0$ for all $\tau$. Q.E.D

Theorem 4.4: $\mu > 0$; i.e., there is a moral hazard agency contracting concern.

Proof: Assume the contrary; i.e., $\mu \leq 0$. Denote by $\hat{t}(\tau)$ the transfer required under the optimum risk sharing condition (4.5) to result in the incomplete information participation shadow price defined by

$\gamma = -\left[ \frac{\Sigma U_\tau - U_\tau}{\Sigma \lambda v'} + \mu \frac{f_\lambda}{f} \right] \forall \tau$

for $\tau \in \{ t|f_\lambda(\tau, \lambda) > 0 \}$

$-\left[ \frac{\Sigma U_\tau - U_\tau}{\Sigma \lambda v'} \right] \leq \gamma = -\left[ \frac{\hat{\Sigma} U_\tau - \hat{U}_\tau}{\hat{\Sigma} \lambda \hat{v}'} \right]$

$\left[ \frac{U_\tau - \Sigma U_\tau}{\Sigma \lambda v'} \right] \leq \left[ \frac{\hat{U}_\tau - \hat{\Sigma} U_\tau}{\hat{\Sigma} \lambda \hat{v}'} \right] \Rightarrow \hat{t}(\tau) \geq t(\tau) \Rightarrow \pi \leq \hat{\pi} \leq \pi \Rightarrow \hat{S} \geq S$

for $\tau \in \{ t|f_\lambda(\tau, \lambda) < 0 \}$
for \( \tau \in (t|f_x(\tau, \lambda) = 0) \)

\[
\begin{bmatrix}
U_\pi - S_i U_\pi \\
S_i v'
\end{bmatrix} \geq \begin{bmatrix}
\hat{U}_\pi - \hat{S_i} \hat{U}_\pi \\
\hat{S_i} \hat{v}'
\end{bmatrix} \Rightarrow \begin{cases}
\hat{t}(\tau) \leq t(\tau) \\
\hat{\pi} \geq \pi \\
\hat{S} \leq S
\end{cases}
\]

Combining these results with Lemma 4.1, we have

\[
\int U f_x \, d\tau \geq \int \hat{U} f_x \, d\tau
\]

Integrating by parts

\[
\int \hat{U} f_x \, d\tau = - \int P \left[ \frac{d\hat{U}}{d\tau} \right] \, d\tau \xrightarrow{\text{Theorem 4.3}} > 0
\]

\[
\therefore \int U f_x \, d\tau > 0
\]

This implies, by the assumption that the agent's problem has an interior solution (4.11), that the coefficient of \( \mu \) must be negative; thus requiring \( \mu > 0 \). This is a contradiction.

\[
\therefore \mu > 0. \quad \text{QED}
\]

**Lemma 4.5:** Given Lemma 4.2 and Theorem 4.4, \( t' > -\frac{S_i}{S_t} \forall \tau \) [a sufficient condition for (4.8) to characterize the solution of the agent's problem]

**Proof:** Recall the expression for \( t' \) under incomplete information (4.14)

\[
t' = -\frac{S_i S \left[ v'(S_i U_{96} - U_{96}) - v'\left(S_i U_{85} - U_{85}\right) + v S_{T_i} U_\pi + \mu \left( v S_i \right)^2 \frac{\partial (f_x)}{\partial \tau} \right]}{S_t \left[ v'(S_i U_{86} - U_{86}) - v'\left(S_i U_{85} - U_{85}\right) - v S_i (S_i U_{86} - U_{86}) + v S_i U_\pi \right]} \forall \tau
\]

This can be rewritten as

\[
t' = -\frac{S_i X + A}{S_i X + B} \forall \tau, \text{ where } \begin{cases}
X = S_i \left[ v'(S_i U_{86} - U_{86}) - v'\left(S_i U_{85} - U_{85}\right) \right] < 0 \\
A = v S_{T_i} U_\pi + \mu \left( v S_i \right)^2 \frac{\partial (f_x)}{\partial \tau} > 0 \\
B = v'\left(S_i U_{86} - S_i (S_i U_{86} - U_{86}) \right) < 0
\end{cases}
\]

\[
t' = -\frac{S_i X + A}{S_i X + B} > -\frac{S_i X + A}{S_i X + B} > -\frac{S_i X + A}{S_i X + B} \forall \tau. \quad \text{QED}
\]
References


